

Math 207 Quiz #8 Answers

1. (a) IVP: $2y'' + 4y' + 10y = 0$, $y(0) = 1$, $y'(0) = 5$

Solutions to the auxiliary equation: $r = -1 \pm 2i$

Solution to the DE: $y = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t)$

Using the initial conditions, the equation of motion is $y = e^{-t} \cos(2t) + 3e^{-t} \sin(2t)$.

Defining $A = \sqrt{1^2 + 3^2} = \sqrt{10}$ and $\tan \phi = \frac{1}{3}$ with ϕ in quadrant I, we have that

$$y = e^{-t} \cos(2t) + 3e^{-t} \sin(2t) \quad \Rightarrow \quad \boxed{y = \sqrt{10} e^{-t} \sin(2t + \phi), \quad \text{with } \phi = \arctan(\frac{1}{3})}$$

(b) The quasiperiod is $\frac{2\pi}{2} = \boxed{\pi \text{ seconds}}$ and the quasifrequency is $\frac{2}{2\pi} = \boxed{\frac{1}{\pi} \text{ cycles/sec.}}$

(c) We are looking for the first time at which $\sqrt{10} e^{-t} \sin(2t + \phi) = 0 \quad \Rightarrow \quad \sin(2t + \phi) = 0$

$$\text{Consider } 2t + \phi = 0 \quad \Rightarrow \quad t = -\frac{1}{2}\phi \approx -0.16088 \text{ sec}$$

Since the period of the function is π , the sine function will equal 0 after every period of $\frac{\pi}{2}$.

So, the first time at which the object will be at equilibrium is $\boxed{t = -\frac{1}{2}\phi + \frac{\pi}{2} \approx 1.4099 \text{ seconds.}}$