Math 207 Quiz #4 Answers

1. This equation is a Bernoulli Equation and can be rewritten as $\dfrac{dy}{dx} - \dfrac{2y}{x} = \dfrac{y^2}{x^2}$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} - \frac{2}{xy} = \frac{1}{x^2} \qquad \text{(Div. by } y^2\text{)}$$

Let $v = \dfrac{1}{y} \quad \Rightarrow \quad \dfrac{dv}{dx} = -\dfrac{1}{y^2}\dfrac{dy}{dx} \quad \Rightarrow \quad -\dfrac{dv}{dx} = \dfrac{1}{y^2}\dfrac{dy}{dx}.$

The equation becomes $-\dfrac{dv}{dx} - \dfrac{2}{x}v = \dfrac{1}{x^2} \quad \Rightarrow \quad \dfrac{dv}{dx} + \dfrac{2}{x}v = -\dfrac{1}{x^2}$

This is a linear equation with an integrating factor of $\mu(x) = x^2$.

Solution: $\quad v = -x^{-1} + Cx^{-2} \quad$ or $\quad v = \dfrac{C-x}{x^2}$

So, $\boxed{y = \dfrac{1}{-x^{-1}+Cx^{-2}} \quad \text{or} \quad y = \dfrac{x^2}{C-x}.}$ Additionally, $y \equiv 0$ is a solution that was lost.

2. Let $A(t)$ = amount of carbon monoxide in the room in ft^3 at t minutes

Since the volume of the room is 1200 ft^3, we have that the percentage of CO in the room compared with the total air is $\dfrac{A}{1200}$.

$$\Rightarrow \quad \frac{dA}{dt} = \text{Input of CO} - \text{Output of CO} = 0 - \frac{A}{1200}(50) \quad \text{in ft}^3\text{/min}$$

We have the following IVP: $\Rightarrow \quad \dfrac{dA}{dt} = -\dfrac{A}{24}, \qquad A(0) = 0.04(1200) = 48 \text{ ft}^3$

This equation is separable or linear with the solution $A(t) = 48e^{-t/24}$. (Or you can see that the solution will be an exponential function with an exponent of $-t/24$.

So, the carbon monoxide will be reduced by half when $24 = 48e^{-t/24} \quad \Rightarrow \quad t = -24\ln(1/2) \text{ or } 24\ln 2$

$\boxed{t \approx 16.6255 \text{ minutes}}$