Math 207 Quiz #2 Solutions

1. (a) This is a linear equation. Rewriting in standard form: $\frac{dy}{dx} + \frac{5}{x}y = \frac{e^{2x}}{x^4}$ (Equation One)

Finding an integrating factor: Consider $\int \frac{5}{x}\,dx = 5\ln|x| + C$

So, we can take $\mu(x) = e^{5\ln x} = x^5$.

Multiplying Equation One by x^5: $x^5\frac{dy}{dx} + 5x^4y = xe^{2x}$

Rewriting the left-side as the derivative (w.r.t. x) of a product of $\mu(x)$ and y: $\frac{d}{dx}[x^5y] = xe^{2x}$

Antidifferentiating both sides with respect to x: $x^5y = \int xe^{2x}\,dx$

$$\Rightarrow \quad x^5y = \tfrac{1}{2}xe^{2x} - \tfrac{1}{4}e^{2x} + C \qquad \text{(Integration by parts)}$$

$$\Rightarrow \quad y = \tfrac{1}{2}x^{-4}e^{2x} - \tfrac{1}{4}x^{-5}e^{2x} + Cx^{-5}$$

Initial Condition (When $x = 1,\ y = 0$): $0 = \frac{1}{2}e^2 - \frac{1}{4}e^2 + C \quad \Rightarrow \quad C = -\frac{1}{4}e^2$

Solution to the IVP: $\boxed{y = \frac{1}{2}x^{-4}e^{2x} - \frac{1}{4}x^{-5}e^{2x} - \frac{1}{4}e^2x^{-5}}$

(b) This equation is not linear (because of the $\cos^2 y$ term). If we rewrite it, we can see that it is separable.

$$\Rightarrow \quad \frac{dy}{dt} = \frac{t^2\cos^2 y}{5t^3+2} \qquad \Rightarrow \qquad \int \sec^2 y\,dy = \int \frac{t^2}{5t^3+2}\,dt$$

Using substitution on the right-side of the equation: $\tan y = \frac{1}{15}\ln|5t^3+2| + C$

Writing the solutions explicitly: $\boxed{y = \arctan(\frac{1}{15}\ln|5t^3+2| + C)}$