Math 207 Quiz #1 Solutions

1. (a) Since $\frac{d^2y}{dt^2} = 2 - C_1 \cos t - C_2 \sin t$, we have that

$$\frac{d^2y}{dt^2} + y \;=\; 2 - C_1 \cos t - C_2 \sin t + t^2 + C_1 \cos t + C_2 \sin t \;=\; t^2 + 2 \quad \text{Yay!}$$

(b) We know from part (a) that the general solution is $y = t^2 + C_1 \cos t + C_2 \sin t$.

We want $y(0) = 0 \quad \Rightarrow \quad 0 = C_1$ and $y'(0) = 4 \quad \Rightarrow \quad 4 = C_2$

So, the particular solution is $\boxed{y = t^2 + 4 \sin t.}$

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2. Implicit differentiation: $\frac{d}{dx}[\sin y + xy] = \frac{d}{dx}[x^2 + 1] \quad \Rightarrow \quad \cos y \frac{dy}{dx} + y + x\frac{dy}{dx} = 2x$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x-y}{\cos y + x} = -\frac{y-2x}{\cos y + x}$$

$\boxed{\text{Yes, it is a solution.}}$

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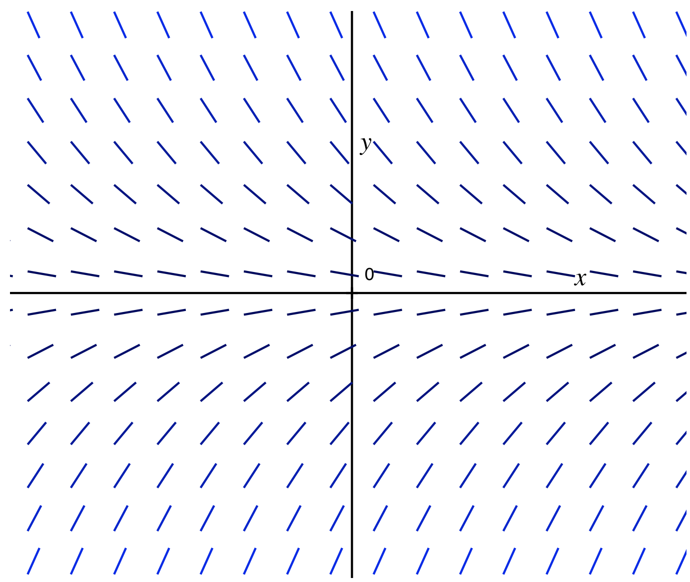
3. Using isoclines: $\frac{dy}{dx} = c \quad \Rightarrow \quad y = -c$

This means that the slope of solution curves is c along the horizontal lines $y = -c$.

The direction field is shown to the right. Note that along the horizontal line $y = 0$, solution will have slope 0, which is hard to see in the figure.



As $x \to \infty$, all solutions approach the x-axis which is the equilibrium solution $y \equiv 0$.

As $x \to -\infty$, solutions above the x-axis will increase without bound and solutions below the x-axis will decrease without bound. The only solution that does not become infinite as $x \to -\infty$ is the solution $y \equiv 0$.

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4. Note: $f(x, y) = 2x + y$

Initial Point: $(0, 3) \quad \Rightarrow \quad x_0 = 0, \quad y_0 = 3$

1st Step: $x_1 = 0 + h = 1, \qquad y_1 = y_0 + h \cdot f(0, 3) = 6 \qquad \Rightarrow \quad$ Point: (1,6)

2nd Step: $x_2 = 1 + h = 2, \qquad y_2 = y_1 + h \cdot f(1, 6) = 14 \qquad \Rightarrow \quad$ Point: (2,14)

So, $\boxed{y(2) \approx 14.}$