Math 207 Topics

Chapter 1: Introduction

- Definition of differential equation §1.1
- Definition of an initial value problem - §1.2
- Existence & Uniqueness Theorem of a first- order DE - §1.2
- Direction Fields (Sketching using Isoclines) - §1.3
- Euler's Method for approximating solutions of $\frac{dy}{dx} = f(x, y)$:
  ($x_{n+1} = x_n + \Delta x, \quad y_{n+1} = y_n + \Delta x f(x_n, y_n)$) - §1.4

Chapter 2: First-Order DEs

- Finding solutions to the following types of equations:
  - Separable Equations: $\frac{dy}{dx} = f(x)g(y)$ - §2.2
  - Linear Equations: $\frac{dy}{dx} + P(x)y = Q(x)$ (Int. Factor $\mu(x) = e^{\int P(x)\,dx}$) - §2.3
  - Exact Equations: $M\ dx + N\ dy = 0$ with $\frac{dM}{dy} = \frac{dN}{dx}$ - §2.4
  - Special Integrating Factors:
    $\mu(x) = \exp[\int \frac{\partial M/\partial y - \partial N/\partial x}{N}\,dx]$ OR $\mu(y) = \exp[\int \frac{\partial N/\partial x - \partial M/\partial y}{M}\,dy]$ - §2.5
  - Homogeneous Equations: $\frac{dy}{dx} = g(\frac{y}{x})$ (Substitute $y = vx \rightarrow$ Separable equation) - §2.6
  - Bernoulli Equations: $\frac{dy}{dx} + P(x)y = Q(x)y^n$
    (Divide by y^n and substitute $v = y^{1-n} \rightarrow$ Linear equation) - S2.6
- Note: You may have lost solutions whenever you are possibly dividing out a solution.

Chapter 3: Mathematical Models Involving First-Order Equations

- $\frac{dx}{dt} =$ (Rate of Increase of x) $-$ (Rate of Decrease of x)
- Compartmental Analysis (Mixing tanks, Population models) - §3.2
- Heating and Cooling - §3.3
  - Newton's Law of Cooling (Rate of change due to outside temperature): $\frac{dT}{dt} = k[M(t) - T(t)]$,
    $1/k =$ Time constant
  - Another possible factor in temperature change: Heating/Air conditioning systems
- Free-Falling Objects: $m\frac{dv}{dt} = mg - bv$ - §3.4
  - Terminal velocity

Chapter 4: Linear Second-Order Equations

- Mass-Spring Oscillator $my'' + by' + ky = F_{ext}$ - §4.1
  - Overdamped or critically damped: $b^2 \geq 4mk$ (Does not oscillate) - §4.9
  - Underdamped: $b^2 < 4mk$ (Oscillates as $t \rightarrow \infty$)

- Homogeneous Linear Equations - §4.2 & 4.3
  - Two real auxiliary equation solutions $r_1,\ r_2$: $y_h = c_1 e^{r_1 t} + c_2 e^{r_2 t}$
  - One real auxiliary equation solution r (Double root): $y_h = c_1 e^{rt} + c_2 t e^{rt}$
  - Complex auxiliary equation solutions $r = \alpha \pm \beta i$ (Double root): $y_h = c_1 e^{\alpha t}\cos(\beta t) + c_2 t e^{\alpha t}\sin(\beta t)$
- Nonhomogeneous Equations: $ay'' + by' + cy = f(t)$ - §4.4 & 4.5
  - General Solution: $y = y_h + y_p$, where y_p is the particular solution that yields $f(t)$ when plugged into $ay'' + by' + cy$. y_p is obtained by using the method of undetermined coefficients. See the box on pg. 193 to determine the form of y_p
  - Superposition Principle - §4.5
- Steady-state solution to $my'' + by' + cy = F_0\cos(\gamma t)$: $y_p = \frac{F_0}{\sqrt{(k-m\gamma^2)^2+b^2\gamma^2}}\sin(\gamma t + \phi)$ with

  $\tan\phi = \frac{k-m\gamma^2}{b\gamma}$ - §4.10
- Resonance frequency of an underdamped system: $\frac{\gamma_r}{2\pi}$ with $\gamma_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$ - §4.10
- Resonance frequency of a system with no damping: $\frac{\gamma_r}{2\pi}$ with $\gamma_r = \omega = \sqrt{\frac{k}{m}}$ - §4.10

Chapter 5: Introduction to Systems and Phase-Plane Analysis

- Substitution Method for solving systems - §5.1 & 5.2
- Elimination Method for solving systems - §5.2
- Interconnected fluid tanks and heat exchange between two rooms - §5.1 & 5.2
- Given a system $\frac{dx}{dt} = f(x,y)$, $\frac{dy}{dt} = g(x,y)$, the phase-plane equation is $\frac{dy}{dx} = \frac{g(x,y)}{f(x,y)}$ - §5.4
  - Integral curves (Solutions to the phase-plane equation)
  - Trajectories (direction of solutions along the integral curves $x(t)$ and $y(t)$ as $t \to \infty$)
  - Critical points (equilibrium solutions)