Math 207
Final Exam
June 9th, 2010

Name: _____

1. Your exam contains 6 questions and 9 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 2 hours for this exam.
3. Make sure to **ALWAYS SHOW YOUR WORK**; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. Put a box around your final answer where applicable.
5. Leave answers in exact form (as simplified as possible).
6. You are allowed one 8.5" $\times$ 11" handwritten notesheet (both sides).
7. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic).
8. If you need extra space, use the back of the exam and clearly indicate this.

Problem	Total Points	Score
1	24	
2	17	
3	13	
4	20	
5	14	
6	12	
Total	100	

1. (24 pts.) Find the general solution to each equation below.

(a) (8 pts.) $x \cdot \dfrac{dy}{dx} + 3y = 4x^2 + 6$ (Write your answer explicitly.)

(b) (8 pts.) $\dfrac{dy}{dx} - xy^2 \sin(x^2 + 3) = 0$ (Write your answer explicitly.)

#1 Continued on the Next Page

#1 Continued:

(c) (8 pts.) $x^2y\ dx\ +\ (2x^3 + 18y^3)dy = 0$ (You may write your answer implicitly.)

2. (17 pts.) Suppose you enter a shed that is initially 80°F. The temperature outside the shed remains constant at 98°F. You are going to work in the shed for many hours so you turn on the air conditioning when you enter. The air conditioner has the capacity to cool the shed by 5°F per hour. The time constant for the shed is 2 hours.

   (a) (14 pts.) Find an equation for the temperature T (in °F) of the shed t hours after you enter.

   (b) (3 pts.) Does the internal temperature of the shed ever reach 90°? If so, when?

3. (13 pts.) Find a general solution of the system $\begin{cases} x' = 3x - y \\ y' = 4x - y + t \end{cases}$ with either substitution or elimination.

(Note: All differentiation is with respect to t. You should have only 2 unknown constants in your solution. Simplify your answers as much as possible.)

4. (20 pts.) A 2-kg mass is attached to a spring with a spring constant of 26 N/m. The damping constant for the system is 8 N-sec/m. Assume there are no external forces on the system.

   At time $t = 0$, the mass is displaced 6 meters from the equilibrium position (in the positive direction) and released.

   (a) (13 pts.) Determine the equation of motion of the mass.

#4 Continued on the Next Page

#4 Continued:

(b) (4 pts.) Find the first time ($t > 0$) at which the mass passes through the equilibrium position.

(c) (3 pts.) Find the quasifrequency of the system and the resonance frequency of the system.

---

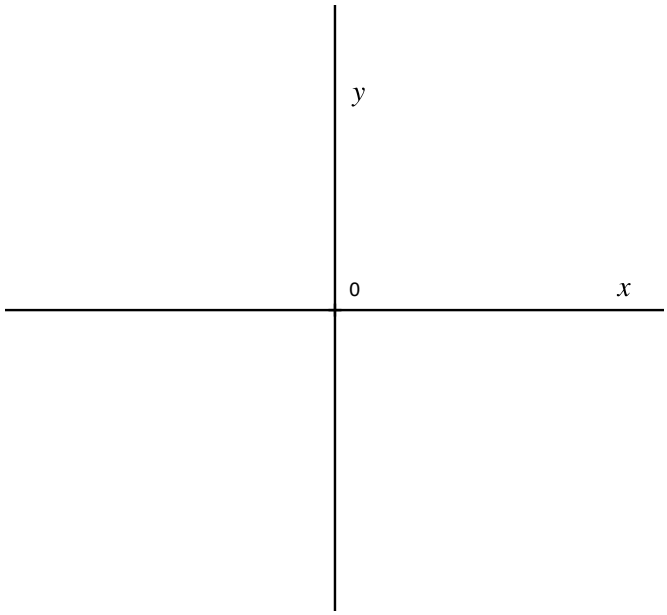
5. (14 pts.) Consider the system of differential equations $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = -2y \end{cases}$

(a) (2 pts.) Determine the critical point set for the system (equilibrium solutions).

#5 Continued on the Next Page

#5 Continued:

(b) (10 pts.) Solve the phase-plane equation for the system $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = -2y \end{cases}$. Then sketch several representative trajectories (with their flow arrows). Write the solution explicitly.



(c) (2 pts.) Consider the solution of the above system with initial values $x(0) = -2$, $y(0) = 4$. Given what you see in the phase-plane, what happens to $x(t)$ and $y(t)$ as $t \to \infty$?

6. (12 pts.) Two tanks, each initially containing 40 liters of pure water, are interconnected by pipes with liquid flowing from tank A into tank B at a rate of 5 liters/minute and from tank B to tank A at a rate of 2 liters/minute. Liquid flows out of tank B at a rate of 3 liters/minute. A brine solution with a concentration of 0.2 kg of salt/liter flows into tank A at a rate of 3 liters/minute. Assume the tanks are well-stirred.

   (a) (10 pts.) Set up, but **do not solve**, a system of differential equations modeling changes to the amount of salt in each tank. Be sure to **define any variables** you use and give **initial conditions.**

   (b) (2 pts.) What will happen to the concentration of salt in each tank as time goes on? (You should be able to answer this without solving the system.)