

**Math 207**  
**Exam 2 Answers**

1. Let  $A(t)$  = Amount of acid in the tank (in gallons) at  $t$  minutes

$$\text{IVP: } \frac{dA}{dt} = 4(0.06) - 2\left(\frac{A}{200+2t}\right) = 0.24 - \frac{A}{100+t}, \quad A(0) = 0.02(200) = 4 \text{ gallons}$$

This is a linear equation with solution  $A = 0.12(100 + t) - \frac{800}{100+t}$

OR  $A = \frac{0.12t^2 + 24t + 400}{100+t}$

2. (a) Let  $P(t)$  = Population of cod at year  $t$

$$\text{IVP: } \frac{dP}{dt} = 3P - 900,000, \quad P(0) = 200,000$$

Note that with the initial condition  $\frac{dP}{dt}$  is negative, which means that the population is decreasing. In fact, if  $P < 300,000$ ,  $\frac{dP}{dt}$  is negative. So, the population will continue to decrease at a faster and faster rate of decrease as time goes on. Eventually, the population will reach 0.

(b) If  $A$  is the fishing rate in fish/year, then  $\frac{dP}{dt} = 3P - A$ . In order for the cod population to remain constant at 200,000, we need  $\frac{dP}{dt} = 3(200,000) - A = 0 \Rightarrow A = \boxed{600,000 \text{ fish/year}}$

3. (a) IVP:  $2y'' + 12y' + 36y = 0, \quad y(0) = 0, \quad y'(0) = 9$

Solutions to the Aux. Equation:  $r = -3 \pm 3i$

$$\text{Solution: } y = c_1 e^{-3t} \cos(3t) + c_2 e^{-3t} \sin(3t)$$

Using initial conditions:  $c_1 = 0, \quad c_2 = 3 \Rightarrow \boxed{y = 3e^{-3t} \sin(3t)}$

(b) The object will oscillate around the equilibrium as  $t \rightarrow \infty$ . The amplitude of the oscillations will approach 0.

4. Solutions to the Aux. Equation:  $r = 0, \quad r = 3$

$$\text{Homogeneous Solution: } y_h = c_1 + c_2 e^{3t}$$

$$\text{Particular Solution Form: } y_p = Ate^{3t} + B \cos t + C \sin t$$

$$\text{Matching Coefficients to Find A, B, and C: } A = \frac{5}{3}, \quad B = -\frac{3}{10}, \quad C = \frac{1}{10}$$

Solution: 
$$y = y_h + y_p = c_1 + c_2 e^{3t} + \frac{5}{3} t e^{3t} - \frac{3}{10} \cos t + \frac{1}{10} \sin t$$

5. Answers may vary. Here's one solution.

Since the general solution to a linear second-order differential equation with constant coefficients can consist of a homogeneous solution and a particular solution, we can look at the terms with the constants as the homogeneous solution and the rest as the particular solution.  
 $\Rightarrow y_h = c_1 e^{2t} + c_2 t e^{2t}$  and  $y_p = 4t^2 - 1$ .

The homogeneous solution points to an auxiliary equation with double root of  $r = 2$ .  
 $(r - 2)^2 = r^2 - 4r + 4 = 0$  is such an equation.

If this is the auxiliary equation, then the differential equation must look like  $y'' - 4y' + 4y = f(t)$ .

To find  $f(t)$ , keep in mind that the particular solution must yield  $f(t)$  when it is plugged into  $y'' - 4y' + 4y$ . Since  $y_p'' - 4y_p' + 4y_p = 16t^2 - 32t + 4$ , we must have that  $f(t) = 16t^2 - 32t + 4$ .

Differential equation: 
$$y'' - 4y' + 4y = 16t^2 - 32t + 4$$