

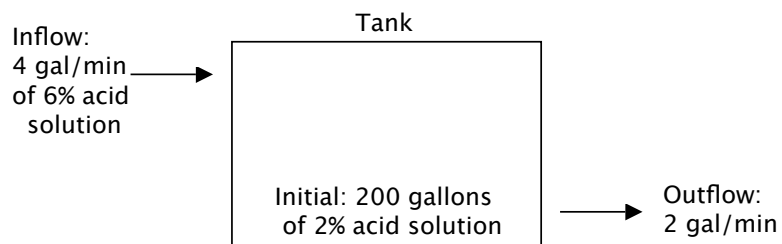
**Math 207**  
**Exam 2**  
**May 20th, 2010**

Name: \_\_\_\_\_

1. Your exam contains 5 questions and 6 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 50 minutes for this exam.
3. Make sure to **ALWAYS SHOW YOUR WORK**; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. Put a box around your final answer where applicable.
5. Leave answers in exact form (as simplified as possible).
6. You are allowed one 3" × 5" notecard (both sides).
7. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic).
8. If you need extra space, use an extra sheet and staple it to the back of the exam and clearly indicate this.
9. **Define all variables you use** when writing differential equations or initial value problems.

Problem	Total Points	Score
1	22	
2	20	
3	22	
4	20	
5	16	
Total	100	

1. (22 pts.) A large tank initially contains 200 gallons of a solution that is 2% acid. A 6% acid solution is pumped into the tank at a rate of 4 gal/min, and the well-stirred mixture flows out of the tank at a rate of 2 gal/min. Find a formula for the amount of acid in the tank (in gallons) at  $t$  minutes.



2. (20 pts.) The population of cod in a certain area is initially 200,000. If left alone, the population would increase at a rate **proportional** to the population, with a proportionality constant of 3/year. However, commercial fishing in the area removes fish at a constant rate of 900,000 fish/year.

(a) (12 pts.) Write an initial value problem modeling the rate of change of the cod population, taking into account all the given information. Use your model to predict what will eventually happen to the cod population. **You do not need to solve the DE to answer this question.**

(b) (8 pts.) In order for the cod population to remain constant at 200,000, what would the commercial fishing rate have to be?

3. (22 pts.) A 2-kg mass is attached to a spring with stiffness 36 N/m. The damping constant for the system is 12 N-sec/m. There are no external forces acting on the system. Initially, the mass is at the equilibrium and the initial velocity is  $y'(0) = 9$  m/s.

(a) (16 pts.) Find the equation of motion of the object.

(b) (6 pts.) **Briefly** describe the motion of the mass as  $t \rightarrow \infty$ .

4. (20 pts.) Find the general solution of  $y'' - 3y' = 5e^{3t} - \sin t$ .

5. (16 pts.) Find a linear second-order differential equation with constant coefficients whose general solution is given by

$$c_1 e^{2t} + c_2 t e^{2t} + 4t^2 - 1,$$

where  $c_1$  and  $c_2$  are arbitrary constants.