Math 152 Quiz #8 Answers

1. (3 pts.) Evaluate $\int t^2\sqrt{9t^2-1}\,dt$ using the following table entry:

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$$

Letting $u = 3t \quad \Rightarrow \quad du = 3\ dt \quad \Rightarrow \quad \frac{1}{3}\ du = dt$, the integral now becomes

$$\int \frac{1}{3}\frac{u^2}{9}\sqrt{u^2-1}\,du = \frac{1}{27}\int u^2\sqrt{u^2-1}\,du \qquad (\text{since } t^2 = \frac{u^2}{9})$$

$$= \frac{1}{27}[\tfrac{u}{8}(2u^2-1)\sqrt{u^2-1} - \tfrac{1}{8}\ln|u+\sqrt{u^2-1}|] + C$$

$$= \boxed{\frac{1}{216}[3t(18t^2-1)\sqrt{9t^2-1} - \ln|3t+\sqrt{9t^2-1}|] + C}$$

2. Since $\Delta x = \frac{5-2}{6} = \frac{1}{2}$, we have that $x_0 = 2$, $x_1 = 2.5$, $x_2 = 3$, $x_3 = 3.5$, $x_4 = 4$, $x_5 = 4.5$, and $x_6 = 5$.

$$\int_2^5 \frac{e^x}{x}\,dx \approx S_6 = \frac{\Delta x}{3}[f(2) + 4f(2.5) + 2f(3) + 4(3.5) + 2f(4) + 4f(4.5) + f(5)]$$

$$= \boxed{\frac{1}{6}[\frac{e^2}{2} + 4\cdot\frac{e^{2.5}}{2.5} + 2\cdot\frac{e^3}{3} + 4\cdot\frac{e^{3.5}}{3.5} + 2\cdot\frac{e^4}{4} + 4\cdot\frac{e^{4.5}}{4.5} + \frac{e^5}{5}]}$$

3. (a) $\lim_{t\to 0}\frac{e^{6t}-1}{t^2}$

Note that as $t \to 0$, the numerator approaches 0 and the denominator approaches 0. So, using L'Hospital's Rule, we have

$$\lim_{t\to 0}\frac{e^{6t}-1}{t^2} = \lim_{t\to 0}\frac{6e^{6t}}{2t}$$

Since this is of the form "$\frac{6}{0}$", we cannot apply L'Hospital's Rule again, but because of its form, we know that the function $\frac{6e^{6t}}{2t}$ is infinite (has a vertical asymptote) at $t = 0$.

So, $\lim_{t\to 0}\frac{6e^{6t}}{2t}$ $\boxed{\text{Does Not Exist*}}$

(*Why not put ∞ or $-\infty$ down? From the right, this function approaches ∞ and from the left, it approaches $-\infty$, so the overall limit cannot be simply written as ∞ or $-\infty$.)

(b) Note that as $\theta \to \pi^-$, the numerator approaches 0 and the denominator approaches 2. So, L'Hospital's Rule does not apply in this case. Instead, since the expression $\frac{\sin\theta}{1-\cos\theta}$ is defined at $\theta = \pi$, we can simply evaluate the expression at the value $\theta = \pi$ to get the value of the limit.

So, $\lim_{\theta\to\pi^-}\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\pi}{1-\cos\pi} = \frac{0}{2} = \boxed{0.}$