

Math 152 Quiz #6 Answers

1. Since $y = \frac{\ln x}{x^2}$ is positive for $1 \leq x \leq e$, the area is given by:

$$\text{Area} = \int_1^e \frac{\ln x}{x^2} dx = \int_1^e x^{-2} \cdot \ln x dx$$

Using integration by parts with $u = \ln x$ and $dv = x^{-2} dx \Rightarrow du = \frac{1}{x} dx$ and $v = -x^{-1}$

$$\begin{aligned} \text{Area} &= \int_1^e x^{-2} \cdot \ln x dx = -x^{-1} \ln x \Big|_1^e - \int_1^e -x^{-2} dx \\ &= -x^{-1} \ln x - x^{-1} \Big|_1^e \\ &= \boxed{-2e^{-1} + 1 \text{ or } -\frac{2}{e} + 1} \end{aligned}$$

2. Trig. Subst.: Let $x = 2 \sin \theta \Rightarrow dx = 2 \cos \theta d\theta$

$$\begin{aligned} \int \frac{5x^3}{\sqrt{4-x^2}} dx &= \int \frac{5(2 \sin \theta)^3}{\sqrt{4-4 \sin^2 \theta}} \cdot 2 \cos \theta d\theta \\ &= \int \frac{40 \sin^3 \theta}{2 \cos \theta} \cdot 2 \cos \theta d\theta \\ &= \int 40 \sin^3 \theta d\theta \\ &= \int 40(1 - \cos^2 \theta) \sin \theta d\theta \\ &= \int -40(1 - u^2) du \quad (\text{Substitution: } u = \cos \theta, \quad du = -\sin \theta d\theta) \\ &= -40(u - \frac{1}{3}u^3) + C \\ &= -40(\cos \theta - \frac{1}{3} \cos^3 \theta) + C \\ &= -40\left(\frac{\sqrt{4-x^2}}{2} - \frac{1}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3\right) + C \quad (\text{since } \cos \theta = \frac{\sqrt{4-x^2}}{2} \text{ using a reference triangle}) \\ &= \boxed{-20\sqrt{4-x^2} + \frac{5}{3}(4-x^2)^{3/2} + C} \end{aligned}$$