# Math 152
## Exam 1 Answers

1. Upper Estimate: (Using the rate values at the beginning of each interval)

$$\text{Total Cheese} = 3(12 + 11 + 10.5 + 10 + 9) = \boxed{157.5 \text{ million pounds}}$$

Lower Estimate: (Using the rate values at the end of each interval)

$$\text{Total Cheese} = 3(11 + 10.5 + 10 + 9 + 8) = \boxed{145.5 \text{ million pounds}}$$

2. (a) The function $g(x)$ is increasing when $f(x)$ is positive. $\Rightarrow$ $\boxed{\text{Intervals: (0,3), (5,6)}}$

(This is due to the fact that $f(x)$ is the derivative of $g(x)$ (FTC 1). Another way to think about it is with net area. The function $g(x)$ will increase when $f(x)$ is positive since at these values, the area under $f(x)$ is counted positively and $g(x)$ is defined in terms of net area.)

(b) (5 pts.) At what $x$-value does $g(x)$ have an absolute maximum?

The absolute maximum will occur at $\boxed{x = 3.}$
Why? As seen in part (a), $g(x)$ is increasing on the intervals (0,3) and (5,6). So, the only possible values at which the absolute maximum can occur is at $x = 6$ and at $x = 3$. Since the area of the region below $f(x)$ between $x = 5$ and $x = 6$ is less than the area of the region between $f(x)$ and the $x$-axis from $x = 3$ to $x = 5$, $g(x)$ decreases more from $x = 3$ to $x = 5$ than it increases from $x = 5$ to $x = 6$.

(c) (5 pts.) Approximate $g'(2)$.

FTC 1 states that $g'(x) = f(x)$. So, $g'(2) = f(2) \approx \boxed{3}$

3. Here's one way to write the sum:

$$\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{4}} + \sqrt{\tfrac{1}{6}} + \sqrt{\tfrac{1}{8}} + \sqrt{\tfrac{1}{10}} + \sqrt{\tfrac{1}{12}} = \boxed{\sum_{i=1}^{5} \sqrt{\frac{1}{2i}}}$$

4. (a) $$\int \cos x(\tan x + 2)\ dx = \int (\sin x + 2\cos x)\ dx \qquad \text{(Distributing } \cos x\text{)}$$

$$= \boxed{-\cos x + 2\sin x + C}$$

(b) $$\int e^x \cdot \sqrt[4]{8e^x - 1}\ dx = \frac{1}{8}\int \sqrt[4]{u}\ du \qquad \text{(Substitution: } u = 8e^x - 1 \qquad du = 8e^x\ dx\text{)}$$

$$= \tfrac{1}{8} \cdot \tfrac{4}{5} u^{5/4} + C$$

$$= \boxed{\tfrac{1}{10}(8e^x - 1)^{5/4} + C}$$

(c) $$\int_e^4 (3 + \frac{1}{t\ln t})\ dt = \int_e^4 3\ dt + \int_e^4 \frac{1}{t\ln t}\ dt$$

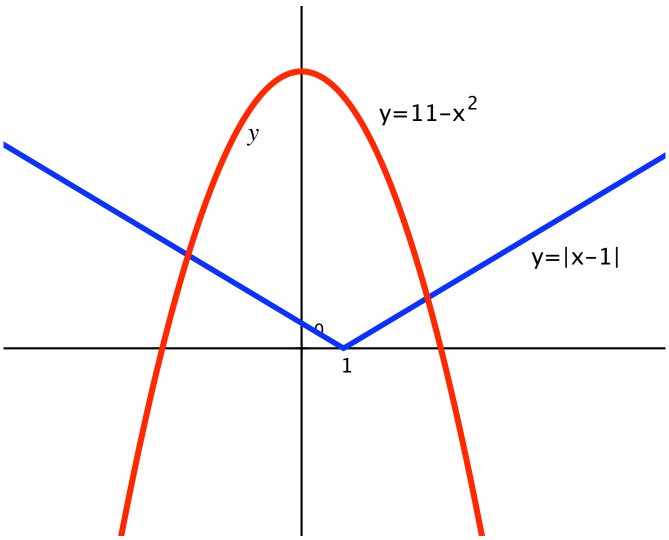
$$= 3t\ |_e^4 + \int \frac{1}{u}\ du \qquad \text{(Substitution on 2nd Integral: } u = \ln t \qquad du = \tfrac{1}{t}\ dt)$$

$$= 3(4-e) + \ln|u|$$

$$= 3(4-e) + \ln|\ln t|\ |_e^4$$

$$= \boxed{3(4-e) + \ln(\ln 4)}$$

5. (20 pts.) Find the area of the region between $y = |x-1|$ and $y = 11 - x^2$ for $x \geq 0$.
(Sketch a rough graph of the region.)




Note: We are only focusing on $x \geq 0$, so we do not need to find both intersections.
To find the intersection we need, solve

$$11 - x^2 = x - 1$$

$$\Rightarrow \quad 0 = x^2 + x - 12$$

$$\Rightarrow \quad 0 = (x+4)(x-3)$$

So the intersection occurs at $x = 3$.

Given that for $x \leq 1$, $|x-1| = 1 - x$ and for $x > 1$, $|x-1| = x - 1$, we have that

$$\text{Area between curves} = \int_0^3 11 - x^2 - |x-1|\ dx$$

$$= \int_0^1 11 - x^2 - (1-x)\ dx + \int_1^3 11 - x^2 - (x-1)\ dx$$

$$= \int_0^1 (10 - x^2 + x)\ dx + \int_1^3 (12 - x^2 - x)\ dx$$

$$= [10x - \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2\ ]_0^1 + [12x - \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\ ]_1^3$$

$$= \boxed{21.5 \text{ units}^2}$$