

Math 152 Quiz #6 Answers

1. Trigonometric Substitution: $x = 2 \sec(\theta)$ $dx = 2 \sec(\theta) \tan(\theta)$

$$\Rightarrow \int \frac{3\sqrt{x^2-4}}{x} dx = 3 \int \frac{\tan^2(\theta)}{\sec(\theta)} = 6 \tan(\theta) - 6\theta + C = 3\sqrt{x^2-4} - 6 \operatorname{arcsec}\left(\frac{x}{2}\right) + C$$

2. Partial Fraction Decomposition: $\frac{2t+12}{9-t^2} = \frac{3}{3-t} + \frac{1}{3+t}$

$$\begin{aligned} \Rightarrow \int_0^2 5 + \frac{2t+12}{9-t^2} dt &= \int_0^2 \left(5 + \frac{3}{3-t} + \frac{1}{3+t}\right) dt = 5t - 3 \ln|3-t| + \ln|3+t| \Big|_0^2 \\ &= 10 + \ln(5) + 2\ln(3) \end{aligned}$$