Math 152 Quiz #3 Solutions

1. (a) Two Ways:

- $$\int 3\sec^2(t)\tan(t)\,dt = \int 3u\,du \qquad (\text{Let } u=\sec(t) \quad \Rightarrow \quad du = \sec(t)\tan(t)\ dt)$$
$$= \tfrac{3}{2}u^2 + C$$
$$= \tfrac{3}{2}\sec^2(t) + C$$

- $$\int 3\sec^2(t)\tan(t)\,dt = \int 3u\,du \qquad (\text{Let } u=\tan(t) \quad \Rightarrow \quad du = \sec^2(t)\ dt)$$
$$= \tfrac{3}{2}u^2 + C$$
$$= \tfrac{3}{2}\tan^2(t) + C$$

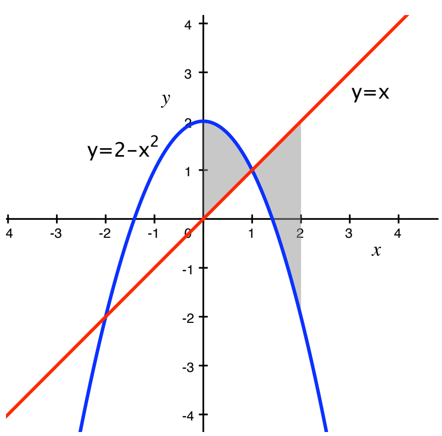
(b) Two Ways:

- $$\int_{-1}^{1} \frac{x^4}{3+x^5}\,dx = \int \frac{1}{5}\cdot\frac{1}{u}\,du \qquad (\text{Let } u = 3+x^5 \quad \Rightarrow \quad du = 5x^4\ dx \quad \Rightarrow \quad \tfrac{1}{5}\ du = x^4\ dx)$$
$$= \tfrac{1}{5}\ln|u|$$
$$= \tfrac{1}{5}\ln|3+x^5| \ \ \Big|_{-1}^{1}$$
$$= \tfrac{1}{5}\ln|3+1^5| - \tfrac{1}{5}\ln|3+(-1)^5|$$
$$= \tfrac{1}{5}\ln(4) - \tfrac{1}{5}\ln(2)$$

- $$\int_{-1}^{1} \frac{x^4}{3+x^5}\,dx = \int_{2}^{4} \frac{1}{5}\cdot\frac{1}{u}\,du \qquad (\text{Let } u = 3+x^5 \quad \Rightarrow \quad du = 5x^4\ dx \quad \Rightarrow \quad \tfrac{1}{5}\ du = x^4\ dx)$$
(Changing bounds: If $x=-1,\ u=2$. If $x=1,\ u=4$.)
$$= \tfrac{1}{5}\ln|u| \ \ \Big|_{2}^{4}$$
$$= \tfrac{1}{5}\ln(4) - \tfrac{1}{5}\ln(2)$$

So, $\displaystyle\int_{-1}^{1} \frac{x^4}{3+x^5}\,dx = \tfrac{1}{5}\ln(4) - \tfrac{1}{5}\ln(2)$ or $\tfrac{1}{5}\ln(2) \leftarrow$ Using the rule $\ln(a) - \ln(b) = \ln(\tfrac{a}{b})$

2. Sketching the functions:



The functions intersect at $x=-2$ and $x=1$.
(Solve $2-x^2 = x$ to find intersections.)

Note that $y = 2-x^2$ is above $y=x$ for $0 \le x \le 1$.

The line $y=x$ is above $y=2-x^2$ for $1 \le x \le 2$.

Area of the shaded region $= \displaystyle\int_0^2 |2-x^2-x|\,dx = \int_0^1 (2-x^2-x)\,dx \ + \ \int_1^2 (x-(2-x^2))\,dx$

$$= \int_0^1 (2 - x^2 - x)\, dx \; + \; \int_1^2 (x - 2 + x^2)\, dx$$

$$= 2x - \tfrac{x^3}{3} - \tfrac{x^2}{2} \;|_0^1 \quad + \quad \tfrac{x^2}{2} - 2x + \tfrac{x^3}{3} \;|_1^2$$

$$= 3$$