

Math 152 Quiz #2 Solutions

1. (a) $g(8) = \int_{-2}^8 f(t) dt = \text{Net Area between } f(t) \text{ and the } t\text{-axis between } t = -2 \text{ and } t = 8$
 $= (\text{Area above the } t\text{-axis}) - (\text{Area below the } t\text{-axis})$
 $= 8 - 4 \quad (\text{Area of trapezoid is 8, Area of triangle is 4})$
 $= 4$

(b) The absolute maximum value of the function $g(x)$ is $g(4) = \int_{-2}^4 f(t) dt = 8$, since $f(t)$ is positive up to $t = 4$ and non-positive for $t \geq 4$.

The function $g(x)$ is constant for $4 < x < 6$ (acquiring no area), and decreases for $6 < x < 8$ (acquiring "negative" area).

2. $\text{Area} = \int_1^4 \left(\frac{1}{3\sqrt{x}} + 2\right) dx = \int_1^4 \left(\frac{1}{3}x^{-1/2} + 2\right) dx = \frac{1}{3} \cdot 2x^{1/2} + 2x \Big|_1^4$
 $= \frac{2}{3}(4)^{1/2} + 2(4) - \left(\frac{2}{3}(1)^{1/2} + 2(1)\right)$
 $= \frac{4}{3} + 8 - \left(\frac{2}{3} + 2\right)$
 $= \frac{20}{3}$

3. $\text{Distance Traveled} = \int_0^5 |8 - 4t| dt = \int_0^2 8 - 4t dt + \int_2^5 4t - 8 dt$ since $v(t) > 0$ for $0 < t < 2$ and
 $v(t) < 0$ for $t > 2$.
(Note: $-(8 - 4t) = 4t - 8$)
 $= [8t - 2t^2]_0^2 + [2t^2 - 8t]_2^5$
 $= [8(2) - 2(2)^2 - (0 - 0)] + [2(5)^2 - 8(5) - (2(2)^2 - 8(2))]$
 $= 26$

So, the swallow has traveled a distance of 26 kilometers.

4. $\int x^2(8x + 1) dx = \int 8x^3 + x^2 dx = \frac{8}{4}x^4 + \frac{1}{3}x^3 + C = 2x^4 + \frac{1}{3}x^3 + C$

5. $\int_{-4}^{-4} \sqrt{16 - x^2} dx = 0$ since the bounds of integration are the equal.