# Math 152
# Final Exam Answers

1. Area $= \int_0^8 3\sin(\sqrt{x})\,dx \;=\; -6\sqrt{x}\cos(\sqrt{x}) + 6\sin(\sqrt{x})\;\Big|_0^8$

$$= -6\sqrt{8}\cos(\sqrt{8}) + 6\sin(\sqrt{8}) \quad \text{square units}$$

(Using substitution ($u = \sqrt{x}$, $2u\,du = dx$) and then integration by parts)

2. (a) The integral $\int_0^{10} v(t)\,dt$ gives the displacement (difference between the final and initial position) of the platypus from time 0 seconds to time 10 seconds. The units of the definite integral are feet.

   (b) $\int_0^{10} v(t)\,dt =$ Net area between $v(t)$ and the $t$-axis from time 0 to time 10
   $= 9 - 8$
   $= 1$ foot

   (The platypus travels 9 feet in the "positive" direction and 8 feet in the "negative" direction. So, the displacement of the platypus is 1 foot.)

   (c) The platypus is furthest away from its starting position during the interval from 4 seconds to 6 seconds. At this time it has gone 9 feet in the "positive" direction. Afterwards, it will travel back towards its starting position.

3. $$\int_4^{\infty} \frac{10}{x^2 - x - 6}\,dx \;=\; \lim_{t\to\infty}\int_4^t \frac{10}{x^2-x-6}\,dx \;=\; \lim_{t\to\infty}\int_4^t \left(\frac{2}{x-3} - \frac{2}{x+2}\right)dx \quad \text{(Partial fractions)}$$

$$= \lim_{t\to\infty}(2\ln|x-3| - 2\ln|x+2|)\;\Big|_4^t$$

$$= \lim_{t\to\infty}\left(2\ln\left|\frac{t-3}{t+2}\right| - 2\ln 1 + 2\ln 6\right)$$

$$= 2\ln 6$$

(To evaluate the limit above, note that $\frac{t-3}{t+2} \to 1$ as $t \to \infty$. So, $2\ln|\frac{t-3}{t+2}| \to 0$.)

This integral is convergent.

4. Area $= \int_{\pi/2}^{2\pi} \frac{1}{2}(6 - 4\sin(\theta))^2\,d\theta = \int_{\pi/2}^{2\pi}(18 - 24\sin(\theta) + 8\sin^2(\theta))\,d\theta$

$$= 22\theta + 24\cos(\theta) - 2\sin(2\theta)\;\Big|_{\pi/2}^{2\pi}$$

$$= 33\pi + 24 \quad \text{square units}$$

(Using the identity $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ to evaluate the integral.)

5. You can use washers or cylindrical shells to find the volume.

   - Using washers:

     At a given value of $x$, the inner radius of each washer is $r = 3$ and the outer radius is $R = 3 + \sqrt{x-2}$.

     $$\text{Volume} = \int_2^6 \pi[(3+\sqrt{x-2})^2 - 3^2]\,dx = \pi \int_2^6 (6\sqrt{x-2} + x - 2)\,dx$$

     $$= \pi[4(x-2)^{3/2} + \tfrac{1}{2}x^2 - 2x]_2^6 = 40\pi \quad \text{cubic units}$$

   - Using shells: (Integrating with respect to $y$ $\longrightarrow$ Using $x = y^2 + 2$ instead of $y = \sqrt{x-2}$)

     For a given value of $y$, the radius of each shell is $r = 3 + y$ and the height of the shell is $h = 6 - (y^2 + 2) = 4 - y^2$.

     $$\text{Volume} = \int_0^2 2\pi(3+y)(4-y^2)\,dy = 2\pi \int_0^2 (12 + 4y - 3y^2 - y^3)\,dy$$

     $$= 2\pi[12y + 2y^2 - y^3 - \tfrac{1}{4}y^4]_0^2 = 40\pi \quad \text{cubic units}$$

6. (a) $\displaystyle\sum_{i=1}^{200} i = \frac{200(201)}{2} = 20,100$

   (b) $\displaystyle\sum_{i=5}^{200} i = 20,100 - (1+2+3+4) = 20,090$

   (This sum is the same as the sum in part (a), except that it is missing the first 4 terms.)

7. $$\int \frac{2\,dx}{(\arcsin x)^3 \cdot \sqrt{1-x^2}} = 2\int \frac{du}{u^3} = -\frac{1}{(\arcsin x)^2} + C \qquad \text{(For some constant } C\text{)}$$

   (Using substitution: $u = \arcsin x$, $du = \frac{1}{\sqrt{1-x^2}}\,dx$)

8. This equation is separable: $\displaystyle\int e^{5y}\,dy = \int \tan^2 x \cdot \sec^4 x\,dx$

   - Integrating the left-side: $\displaystyle\int e^{5y}\,dy = \frac{1}{5}e^{5y} + C$

   - Integrating the right-side: $$\int \tan^2 x \cdot \sec^4 x\,dx = \int \tan^2 x(\tan^2 x + 1)\sec^2 x\,dx$$
     $$= \int (u^4 + u^2)\,du$$
     $$= \tfrac{1}{5}\tan^5 x + \tfrac{1}{3}\tan^3 x + C$$

   (Using substitution: $u = \tan x$, $du = \sec^2 x\,dx$)

So, we have that $\frac{1}{5}e^{5y} = \frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$ (For some constant $C$)

Solving for $y$: $y = \frac{1}{5}\ln(\tan^5 x + \frac{5}{3}\tan^3 x + C^*)$ (For some constant $C^*$)

Initial condition: Evaluating at $x = 0,\ y = 1$ gives that $C^* = e^5$.

$\Rightarrow$ Particular solution: $y = \frac{1}{5}\ln(\tan^5 x + \frac{5}{3}\tan^3 x + e^5)$