

Math 152
Exam 2 Answers

1. **Using Disks:** $V = \int_1^e \pi(5 \ln(x))^2 dx = 25\pi \int_1^e (\ln(x))^2 dx$

$$= 25\pi [x(\ln(x))^2 - 2x \ln(x) + 2x]_1^e$$

$$= 25\pi(e - 2)$$

(To evaluate integral: Use int. by parts $u = (\ln(x))^2$, $du = \frac{2 \ln(x)}{x} dx$, $dv = dx$, $v = x$)

Using Shells: $V = \int_0^5 2\pi y \cdot (e - e^{y/5}) dy = \int_0^5 2\pi e y dy - \int_0^5 2\pi y \cdot e^{y/5} dy$

$$= 2\pi [\frac{1}{2} e y^2 - 5y e^{y/5} + 25e^{y/5}]_0^5$$

$$= 25\pi(e - 2)$$

(To evaluate the 2nd integral: Use int. by parts $u = y$, $du = dy$, $dv = e^{y/5} dx$, $v = 5e^{y/5}$)

2. $v_{ave} = \frac{1}{5} \int_0^5 (2 + \frac{3t^2 + 12t + 13}{t^2 + 4t + 3}) dt = \frac{1}{5} \int_0^5 (2 + 3 + \frac{4}{t^2 + 4t + 3}) dt$ (Using polynomial division)

$$= \frac{1}{5} \int_0^5 (5 + \frac{2}{t+1} - \frac{2}{t+3}) dt$$
 (Partial fraction decomp.)
$$= \frac{1}{5} [5t + 2 \ln |t+1| - 2 \ln |t+3|]_0^5$$

$$= \frac{1}{5} [25 + 2 \ln(6) - 2 \ln(8) + 2 \ln(3)] \quad \text{OR} \quad 5 + \frac{2}{5} \ln(\frac{9}{4})$$

The units are meters/second.

3. (a) $\int (\frac{e^{2x}}{\sqrt{4 - e^{2x}}} + x) dx = \int \frac{e^{2x}}{\sqrt{4 - e^{2x}}} dx + \int x dx$

$$= -\sqrt{4 - e^{2x}} + x^2 + C$$

(To evaluate the 1st integral: Use substitution with $u = 4 - e^{2x}$, $du = -2e^{2x} dx$)

(b) $\int \frac{\tan^{10}(t)}{7 \sec^{13}(t)} dt = \frac{1}{7} \int \sin^{10}(t) \cdot \cos^3(t) dt$

$$= \frac{1}{7} \int \sin^{10}(t)(1 - \sin^2(t)) \cdot \cos(t) dt$$

$$= \frac{1}{7} [\frac{1}{11} \sin^{11}(t) - \frac{1}{13} \sin^{13}(t)] + C$$

(To evaluate the integral: Use substitution with $u = \sin(t)$, $du = \cos(t) dt$)