

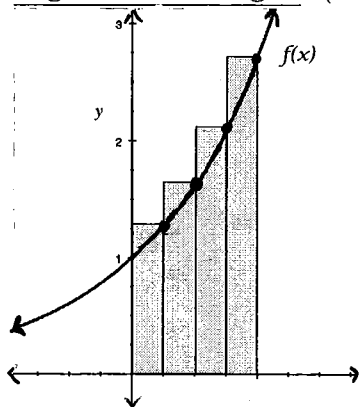
$A = \text{Area under the curve}$   
 $f(x) = e^x$  between  
 $x = 0$  and  $x = 1$

Exact area:  $A = e - 1 \approx 1.71828$

**Things to Note:**

- 1) More Rectangles  $\Rightarrow$  More Work  $\Rightarrow$  Better Estimate
- 2) You can use either left-end rectangles or right-end rectangles.  
With more rectangles, both methods will approach the exact area.

**Right-end rectangles:** (Note: These approximations for the area are all overestimates.)



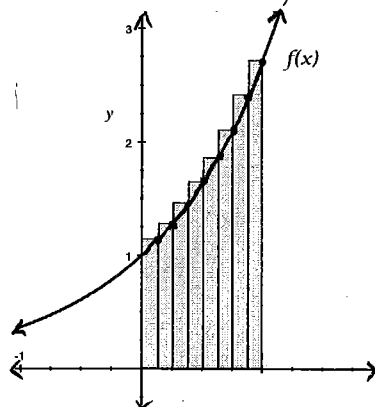
Using 4 right-end rectangles:

$$A \approx R_4 = .25f(.25) + .25f(.5) + .25f(.75) + .25f(1)$$

$$= .25[f(.25) + f(.5) + f(.75) + f(1)]$$

$$= .25[e^{.25} + e^{.5} + e^{.75} + e^1]$$

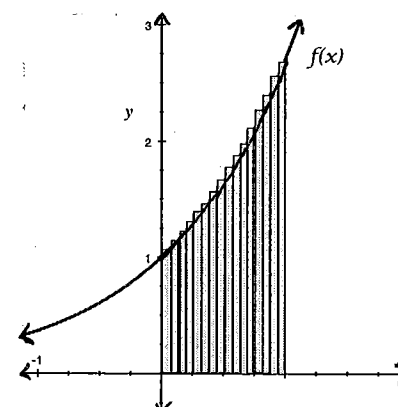
$$\approx 1.942$$



Using 8 right-end rectangles:

$$A \approx R_8 = .125[e^{.125} + e^{.25} + e^{.375} + e^{.5} + e^{.675} + e^{.75} + e^{.875} + e^1]$$

$$\approx 1.8279$$

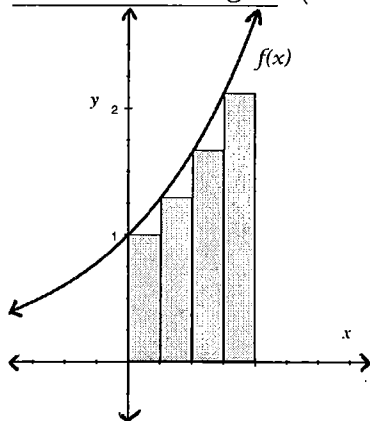


Using 16 right-end rectangles:

$$A \approx R_{16} \approx 1.7725$$

(Lots of work!!!)

**Left-end rectangles:** (Note: These approximations for the area are all underestimates.)



Using 4 left-end rectangles:

$$A \approx L_4 = .25f(0) + .25f(.25) + .25f(.5) + .25f(.75)$$

$$= .25[f(0) + f(.25) + f(.5) + f(.75)]$$

$$= .25[e^0 + e^{.25} + e^{.5} + e^{.75}]$$

$$\approx 1.5124$$

Using 8 left-end rectangles:

$$A \approx L_8 = .125[e^0 + e^{.125} + e^{.25} + e^{.375} + e^{.5} + e^{.675} + e^{.75} + e^{.875}]$$

$$\approx 1.6251$$

Using 16 left-end rectangles:

$$A \approx L_{16} \approx 1.6651$$