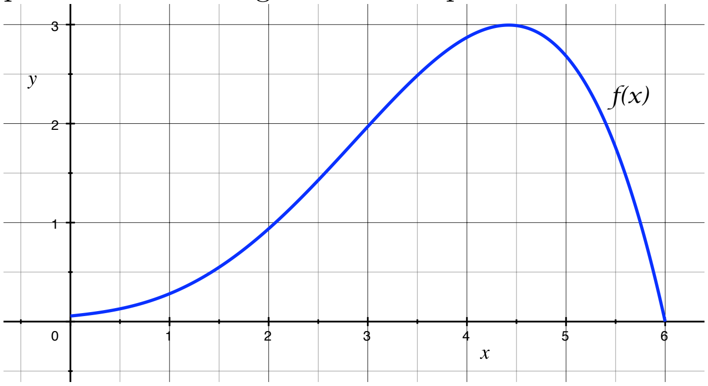
Math 125 Worksheet #9 Solutions

1. Approximate the area under the curve of $f(x)$ as shown below using trapezoidal approximation and again with Simpson's rule with $n = 6$.



Note: $n = 6 \rightarrow \Delta x = \frac{6-0}{6} = 1$

Trapezoid Rule:

$$\int_0^6 f(x)\,dx \approx T_6 = \tfrac{\Delta x}{2}[f(0) + 2f(1) + 2f(2) + 2f(3) + 2f(4) + 2f(5) + f(6)]$$

$$\approx \tfrac{1}{2}[.1 + 2(.3) + 2(.9) + 2(2) + 2(2.8) + 2(2.7) + 0]$$

$$= \frac{35}{4} \quad \text{(Answers will vary.)}$$

Simpson's Rule:

$$\int_0^6 f(x)\,dx \approx S_6 = \tfrac{\Delta x}{3}[f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)]$$

$$\approx \tfrac{1}{3}[.1 + 4(.3) + 2(.9) + 4(2) + 2(2.8) + 4(2.7) + 0]$$

$$= \frac{55}{6} \quad \text{(Answers will vary.)}$$

2. Evaluate the following limits.

   (a) $\displaystyle\lim_{t\to 0} \frac{1-\cos^2 t}{2t^2}$

   Note that this is an indeterminate form of the type "0/0". Since the numerator and denominator are differentiable and derivative of $2t^2$ is not zero **near** $t = 0$, we can use L'Hospital's Rule.

   So, $\displaystyle\lim_{t\to 0} \frac{1-\cos^2 t}{2t^2} = \lim_{t\to 0} \frac{2\cos t \sin t}{4t}$

   Again, this is an indeterminate form of the type "0/0" and we satisfy the hypotheses to use L'Hospital's Rule.

$$\text{So, } \lim_{t\to 0}\frac{1-\cos^2 t}{2t^2} = \lim_{t\to 0}\frac{2\cos t\,\sin t}{4t} = \lim_{t\to 0}\frac{-2\sin^2 t + 2\cos^2 t}{4}$$

$$= \frac{2}{4} \quad \text{(Evaluating expression at } t = 0 \text{ since it is continuous there.)}$$

$$= \frac{1}{2}.$$

(b) $\displaystyle\lim_{x\to\infty}\frac{\ln\left(\frac{1}{x}\right)}{x}$

Note that this is an indeterminate form of the type "$-\infty/\infty$". Since the numerator and denominator are differentiable and derivative of x is not zero, we can use L'Hospital's Rule.

$$\text{So, } \lim_{x\to\infty}\frac{\ln\left(\frac{1}{x}\right)}{x} = \lim_{x\to\infty}\frac{\frac{1}{1/x}\cdot\frac{-1}{x^2}}{1} = \lim_{x\to\infty} -\frac{1}{x} = 0.$$

(c) $\displaystyle\lim_{x\to\infty} x\ln\left(\frac{1}{x}\right)$

Note that this is not an indeterminate form since as $x \to \infty$, $\ln(\frac{1}{x}) \to -\infty$ since $\frac{1}{x} \to 0$.

$$\text{So, } \lim_{x\to\infty} x\ln\left(\frac{1}{x}\right) = -\infty.$$

(d) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{\sin x}\right)^x$

Let $y = \left(\frac{1}{\sin x}\right)^x$. Then $\ln y = x\ln\left(\frac{1}{\sin x}\right) = -x\ln(\sin x)$.

Consider $\displaystyle\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+} -x\ln(\sin x) = \lim_{x\to 0^+} -\frac{\ln(\sin x)}{\frac{1}{x}}$. This is an indeterminate form of the type "$-\infty/\infty$" that satisfies the hypotheses of L'Hospital's Rule.

$$\text{So, } \lim_{x\to 0^+}\ln y = \lim_{x\to 0^+} -\frac{\ln(\sin x)}{\frac{1}{x}} = \lim_{x\to 0^+} -\frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+}\frac{x^2\cos x}{\sin x}$$

This is an indeterminate form of the type "0/0" that satisfies the hypotheses of L'Hospitals Rule.

$$\text{So, } \lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{x^2\cos x}{\sin x} = \lim_{x\to 0^+}\frac{2x\cos x - x^2\sin x}{\cos x}$$

$$= \frac{0}{1} = 0 \quad \text{(Evaluating at } x = 0 \text{ since it is continuous there.)}$$

$$\text{Thus } \lim_{x\to 0^+}\left(\frac{1}{\sin x}\right)^x = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^0 = 1.$$