

Math 125 Worksheet #8 Solutions

1. Evaluate the following integrals.

$$(a) \int \frac{x-7}{x^2+5x+6} dx$$

$$\text{Note: } x^2 + 5x + 6 = (x+3)(x+2)$$

$$\text{So, } \frac{x-7}{x^2+5x+6} = \frac{A}{x+3} + \frac{B}{x+2}.$$

$$\text{Multiplying both sides by } x^2 + 5x + 6: \quad x - 7 = A(x+2) + B(x+3)$$

Here are two ways to solve for A and B :

- Equating coefficients: $x - 7 = Ax + 2A + Bx + 3B$

$$\Rightarrow A + B = 1 \quad \text{and} \quad 2A + 3B = -7$$

Solving for A and B , we have that $A = -9$ and $B = 10$.

- Creating zeros: $\text{Let } x = -2 \Rightarrow -2 - 7 = B(-2 + 3) \Rightarrow B = -9$
 $\text{Let } x = -3 \Rightarrow -3 - 7 = A(-3 + 2) \Rightarrow A = 10$

$$\text{So, } \frac{x-7}{x^2+5x+6} = \frac{10}{x+3} + \frac{-9}{x+2}.$$

$$\begin{aligned} \Rightarrow \int \frac{x-7}{x^2+5x+6} dx &= \int \frac{10}{x+3} + \frac{-9}{x+2} dx \\ &= 10\ln|x+3| - 9\ln|x+2| + C \end{aligned}$$

$$(b) \int \frac{x^2 - 2x + 4}{x^3 - 4x^2 + 4x} dx$$

$$\text{Note: } x^3 - 4x^2 + 4x = x(x-2)^2$$

$$\text{So, } \frac{x^2-2x+4}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}.$$

Multiplying both sides by $x^3 - 4x^2 + 4x$:

$$x^2 - 2x + 4 = A(x-2)^2 + Bx(x-2) + Cx$$

Here are two ways to solve for A , B , and C :

- Equating coefficients:

$$\Rightarrow A + B = 1, \quad -4A - 2B + C = -2, \quad \text{and} \quad 4A = 4$$

Solving for A , B , and C , we have that $A = 1$, $B = 0$, and $C = 2$.

- Creating zeros: Let $x = 2 \Rightarrow 4 - 4 + 4 = 2C \Rightarrow C = 2$
 Let $x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$

To find B , we can plug in another value of x or equate coefficients.

$$\text{Let } x = 1 \Rightarrow 1 - 2 + 4 = 1(1 - 2)^2 + B(1 - 2) + 2 \Rightarrow B = 0$$

$$\text{So, } \frac{x^2 - 2x + 4}{x^3 - 4x^2 + 4x} = \frac{1}{x} + \frac{2}{(x-2)^2}.$$

$$\begin{aligned} \Rightarrow \int \frac{x^2 - 2x + 4}{x^3 - 4x^2 + 4x} dx &= \int \frac{1}{x} + \frac{2}{(x-2)^2} dx \\ &= \ln|x| - \frac{2}{x-2} + C \end{aligned}$$

(c) $\int \frac{x^3}{x-2} dx$

Since the degree of the numerator is greater than the degree of the denominator, we must use polynomial division to simplify the integrand.

$$\begin{array}{r} x-2 \overline{) x^3} \\ \underline{-(x^3 - 2x^2)} \\ 2x^2 \\ \underline{-(2x^2 - 4x)} \\ 4x \\ \underline{-(4x - 8)} \\ 8 \end{array}$$

$$\text{So, } \frac{x^3}{x-2} = x^2 + 2x + 4 + \frac{8}{x-2}.$$

$$\begin{aligned} \Rightarrow \int \frac{x^3}{x-2} dx &= \int x^2 + 2x + 4 + \frac{8}{x-2} dx \\ &= \frac{1}{3}x^3 + x^2 + 4x + 8\ln|x-2| + C \end{aligned}$$