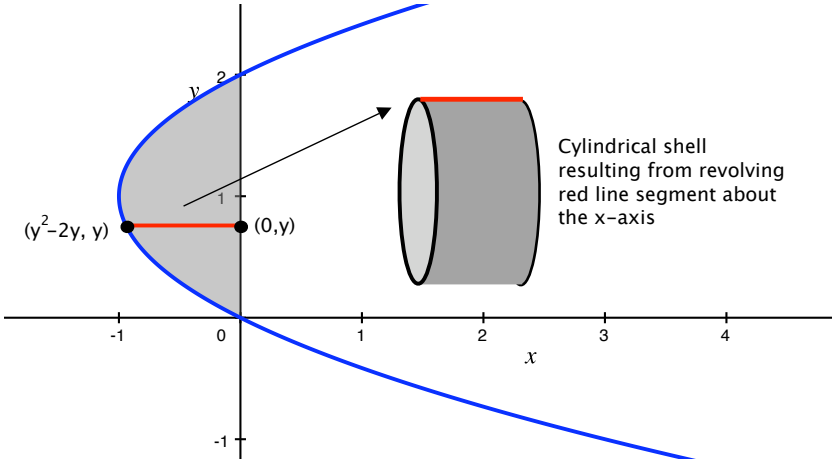
Math 125 Worksheet #5 Solutions

1. Let R = the region bounded by $x = y^2 - 2y$ and the y-axis. Find the volume of the solid obtained by revolving R about the x-axis using cylindrical shells.



For a given value y, the cylindrical shells have radius $r = y$ and height $h = 0 - (y^2 - 2y) = -y^2 + 2y$.

So, the volume of the solid is given by

$$\int_0^2 2\pi y(-y^2 + 2y)\,dy = 2\pi \int_0^2 -y^3 + 2y^2\,dy$$

$$= 2\pi[-\tfrac{1}{4}y^4 + \tfrac{2}{3}y^3]_0^2$$

$$= \tfrac{8}{3}\pi$$

2. Find the average value f_{ave} of the function $f(x) = (x+1)^2$ on the interval $[-1, 2]$. Find a value c such that $f(c) = f_{ave}$.

The average value $= f_{ave} = \frac{1}{2-(-1)}\int_{-1}^2 (x+1)^2\,dx$

$= \frac{1}{3}[\frac{1}{3}(x+1)^3]_{-1}^2$ (Using substitution $u = x + 1$ to integrate.)

$= \frac{1}{3}(9) = 3$

To find c such that $f(c) = f_{ave}$: $(c+1)^2 = 3$

$\Rightarrow \quad c + 1 = \pm\sqrt{3} \quad \Rightarrow \quad c = -1 \pm \sqrt{3}$

3. Find the average value of the function $g(x) = \dfrac{2\sin x}{1+\cos^2 x}$ on the interval $[0, \pi]$.

The average value $= f_{ave} = \frac{1}{\pi - 0}\int_0^{\pi} \frac{2\sin x}{1+\cos^2 x}\,dx$

$= \frac{1}{\pi}\int_1^{-1} \frac{-1}{1+u^2}\,du$ (Using substitution $u = \cos x$.)

$= -\frac{2}{\pi}\ \arctan u\ |_1^{-1}$

$= \frac{2}{\pi}\ \arctan u\ |_{-1}^{1}$

$= \frac{2}{\pi}\ [\arctan(1)\text{-}\arctan(\text{-}1)]$

$= \frac{2}{\pi}\ [\frac{\pi}{4} - (-\frac{\pi}{4})]$

$= 1$