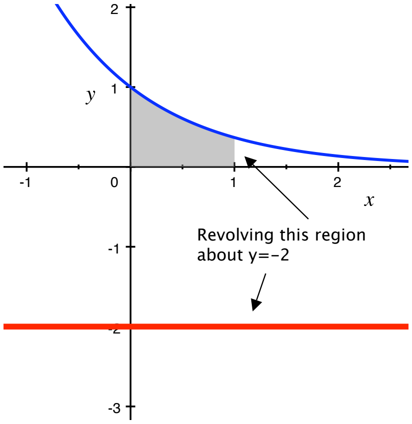
Math 125 Worksheet #4 Solutions

1. Let R = the region bounded by $y = e^{-x}$, the y-axis, and $x = 1$. What is the volume of the solid obtained by revolving R about the line $y = -2$?

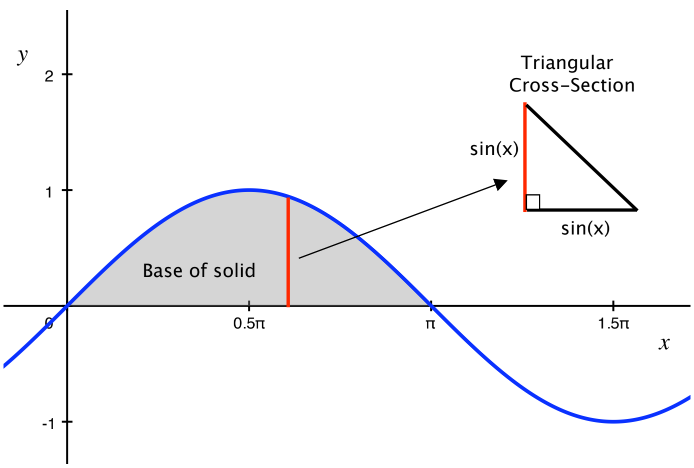


After revolving, the cross-sections perpendicular to the x-axis will be washers. At a given value x, the area of each washer will be $A(x) = \pi(r_1^2 - r_2^2) = \pi((2+e^{-x})^2 - 2^2)$
$= \pi(4e^{-x} + e^{-2x})$.

So, the volume of the solid $= \int_0^1 A(x)\,dx = \int_0^1 \pi(4e^{-x} + e^{-2x})$

$$= \pi[-4e^{-x} - \tfrac{1}{2}e^{-2x}]_0^1$$
$$= -\pi(4e^{-1} + \tfrac{1}{2}e^{-2} - \tfrac{9}{2})$$
$$\approx 9.30167$$

2. The base of a solid S is the region bounded by $y = \sin(x)$ and the x-axis for $0 \le x \le \pi$. Cross-sections of S perpendicular to the x-axis are isosceles right triangles. The right angle of each triangle is on the x axis. What is the volume of S? (Fun trigonometric fact of the day: $\sin^2(x) = \frac{1-\cos(2x)}{2}$.)



The length of the base and the height are both $\sin(x)$ since the triangle is isosceles with right angle on the x-axis.

So, $A(x) = \frac{1}{2}\sin(x)\sin(x)$
$= \frac{1}{2}\sin^2(x)$

So, the volume of the solid $= \int_0^\pi A(x)\,dx = \int_0^\pi \frac{1}{2}\sin^2(x)\,dx$

$$= \tfrac{1}{2}\int_0^\pi \frac{1-\cos(2x)}{2}\,dx$$
$$= \tfrac{1}{4}\int_0^\pi 1 - \cos(2x)\,dx$$
$$= \tfrac{1}{4}[x - \tfrac{1}{2}\sin(2x)]_0^\pi = \frac{\pi}{4}$$