Math 125 Worksheet #3
July 10, 2007

1. Evaluate the following integrals using substitution.

   (a) $\int 5t\,\sin(t^2+2)\,dt$

   Let $u = t^2 + 2 \quad \Rightarrow \quad du = 2t\;dt$ or $\frac{1}{2}\;du = t\;dt$.

$$\begin{aligned}\int 5t\,\sin(t^2+2)\,dt &= 5\int \sin(t^2+2)\cdot t\,dt\\ &= 5\int \sin\,u\cdot\frac{1}{2}\,du\\ &= \tfrac{5}{2}\int \sin\,u\,du\\ &= -\tfrac{5}{2}\cos\,u + C \quad \text{for an arbitrary constant } C\\ &= -\tfrac{5}{2}\cos(t^2+2)+C\end{aligned}$$

   (b) $\int_1^4 e^{\sqrt{x}}\,dx$

   Let $u = \sqrt{x} \quad \Rightarrow \quad du = \frac{1}{2\sqrt{x}}\;dx$ or $2\sqrt{x}\;du = dx \quad \Rightarrow \quad 2u\;du = dx$.

$$\int_1^4 e^{\sqrt{x}}\,dx = \int_1^2 e^u\cdot 2u\,du = 2\int_1^2 ue^u\,du$$

   (When $x=1,\; u=\sqrt{1}=1$ and when $x=4,\; u=2$.)

   Unfortunately, while this is a beautiful integral, we cannot yet evaluate it. I mistakenly added this problem.

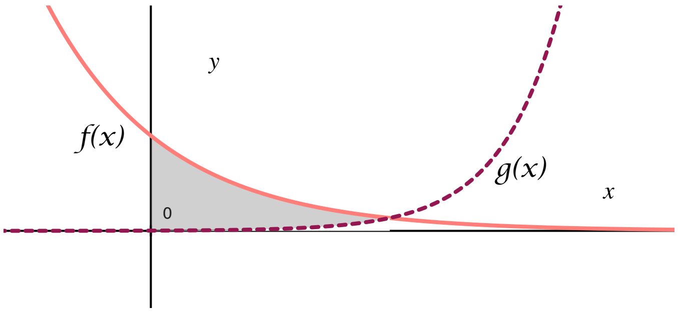
   (c) $\int_{\pi/6}^{\pi/2} 2\,\ln(\sin\,\theta)\,\cot\,\theta\,d\theta$

   Let $u = \ln(\sin\,\theta) \quad \Rightarrow \quad du = \frac{1}{\sin\,\theta}\cdot\cos\,\theta\;d\theta = \cot\,\theta\;d\theta$.

$$\int_{\pi/6}^{\pi/2} 2\,\ln(\sin\,\theta)\,\cot\,\theta\,d\theta = \int_{\ln(\frac{1}{2})}^{0} 2u\,du = u^2\,\Big|_{\ln(\frac{1}{2})}^{0} = 0 - [\ln(\frac{1}{2})]^2 \approx -0.48045$$

   (When $\theta = \frac{\pi}{6},\; u = \ln(\sin(\frac{\pi}{6})) = \ln(\frac{1}{2})$ and when $\theta = \frac{\pi}{2},\; u = \ln(\sin(\frac{\pi}{2})) = \ln(1) = 0$.)

2. For $f(x) = e^{-x}$ and $g(x) = e^{2x-6}$, find the area of the region bounded between the two curves and the y-axis as shown in the figure below.



To find the point of intersection, we must find the x-value for which $e^{-x} = e^{2x-6}$

$$\Rightarrow \quad \ln(e^{-x}) = \ln(e^{2x-6})$$

$$-x = 2x - 6 \quad \Rightarrow \quad x = 2$$

Since $e^{-x} \geq e^{2x-6}$ for $0 \leq x \leq 2$, the area of the region $= \displaystyle\int_0^2 e^{-x} - e^{2x-6}\,dx$.

Note: You can split up the integrals

$$\int_0^2 e^{-x} - e^{2x-6}\,dx = \int_0^2 e^{-x}\,dx - \int_0^2 e^{2x-6}\,dx$$

$$= -e^{-x}|_0^2 - \tfrac{1}{2}e^{2x-6}|_0^2$$

$$= -e^{-2} - (-e^0) - [\tfrac{1}{2}e^{-2} - \tfrac{1}{2}e^{-6}]$$

$$= -\tfrac{3}{2}e^{-2} + 1 + \tfrac{1}{2}e^{-6} \approx 0.7982$$

(You can use substitution for both integrals: $u_1 = -x$ and $u_2 = 2x - 6$)