

Math 125 Final Exam Practice Problem Answers

2. Evaluate the following integrals.

$$(a) \int \frac{y^3 - 4y + 7}{y^2 + 2y - 3} dy = \frac{1}{2}y^2 - 2y + 2\ln|y + 3| + \ln|y - 1| + C$$

(Partial Fractions with polynomial long division)

$$(b) \int_0^8 \frac{dx}{\sqrt[3]{x-8}} = -6$$

(Improper Integral: Undefined at $x = 8$)

$$(c) \int_2^\infty \frac{3}{x^2 - x} dx = 3\ln 2$$

(Improper Integral, Partial Fractions, l'Hospital's Rule)

$$(d) \int \frac{t + 7}{\sqrt{5-t}} dt = -24\sqrt{5-t} + \frac{2}{3}(5-t)^{3/2} + C$$

(Rationalizing Substitution: $u = \sqrt{5-t}$)

$$(e) \int_2^\infty \frac{dx}{x(\ln x)^2} = \frac{1}{\ln 2}$$

(Improper Integral, Substitution $u = \ln x$)

3. Evaluate the following limits.

$$(a) \lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{x} = 0$$

$$(b) \lim_{t \rightarrow 0^+} \sin(t)\ln(t) = 0$$

$$(c) \lim_{x \rightarrow \infty} (e^x + 5x)^{1/x} = e$$

4. Approximate the integral $\int_2^5 3x^2 \ln x dx$ using (a) the Trapezoid Rule and (b) Simpson's Rule with $n = 6$. (Round to four decimal places.) What is the exact value of the integral?

$$\text{Trapezoid Rule: } \int_2^5 3x^2 \ln x dx \approx 157.6548$$

$$\text{Simpson's Rule: } \int_2^5 3x^2 \ln x dx \approx 156.6340$$

$$\text{Exact Value: } \int_2^5 3x^2 \ln x dx = 125 \ln 5 - 8 \ln 2 - \frac{117}{3} \approx 157.6346$$