

## Math 125 Exam #2 Practice Problems Answers

2. Evaluate the following integrals. (There may be more than one way to evaluate.)

(a)  $\int \sin x \ln(\sec x) dx = -\cos x \ln(\sec x) - \cos x + C$   
(Using integration by parts with  $u = \ln(\sec x)$ ,  $dv = \sin x dx$ )

(b)  $\int \frac{t}{\sqrt{t^2 - 6t + 5}} dt = 3\ln\left|\frac{t-3}{2}\right| + \frac{\sqrt{t^2 - 6t + 5}}{2} + \sqrt{t^2 - 6t + 5} + C$   
(Completing the square  $t^2 - 6t + 5 = (t - 3)^2 - 4$  and using trigonometric substitution with  $t - 3 = 2\sec \theta$ )

(c)  $\int_0^{3\pi^2} \cos\sqrt{x + \pi^2} dx = 4$   
(Using substitution with  $w = \sqrt{x + \pi^2} \rightarrow 2w dw = dx$ , and then integration by parts)

(d)  $\int 4\theta \sec^2(2\theta) d\theta = 2\theta \tan(2\theta) - \ln|\sec(2\theta)| + C$   
(Using integration by parts with  $u = 4\theta$ ,  $dv = \sec^2\theta d\theta$   
Can also use substitution  $w = 2\theta$  to simplify integral at beginning.)

(e)  $\int \sin^3 x \cos^7 x dx = \frac{1}{10}\cos^{10}x - \frac{1}{8}\cos^8x + C$   
(Using  $\sin^2x = 1 - \cos^2x$  and substitution  $u = \cos x$ )

(f)  $\int \frac{dx}{(4 - x^2)^{3/2}} = \frac{x}{4\sqrt{4-x^2}} + C$   
(Using trigonometric substitution  $x = 2\sin \theta d\theta$ )

(g)  $\int x^3 e^{x^2} dx = \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C$   
(Using substitution with  $w = x^2 \rightarrow dw = 2x dx$ , and then integration by parts)

(h)  $\int \frac{x^3}{\sqrt{x^2 + 4}} dx = \frac{1}{3}(x^2 + 4)^{3/2} - 4\sqrt{x^2 + 4} + C$   
(Using substitution  $u = x^2 + 4$  OR the trigonometric substitution  $x = 2\tan \theta$ )

3. Let  $R$  be the region bounded by the curve  $y = \sqrt[3]{x+1}$  and the  $x$ -axis for  $0 \leq x \leq 7$ . Find the volume of the solid obtained by revolving  $R$  about the  $y$ -axis. For extra practice, you may want to try to do this problem using washers and again using shells.

Using Shells: Volume =  $\int_0^7 2\pi x \sqrt[3]{x+1} dx = \frac{1209\pi}{14}$

Using Washers: Volume =  $\int_1^2 \pi(7^2 - (y^3 - 1)^2) dy + \int_0^1 \pi(7^2) dy = \frac{1209\pi}{14}$

4. Find the average value of  $f(x) = \sin^{-1}x$  on the interval  $[\frac{1}{2}, 1]$ .

$$f_{ave} = \frac{5\pi}{6} - \sqrt{3} \approx .88594$$

5. Let  $R$  be the region bounded by  $y = e^{2-x}$ , the line  $y = 1$ , and the  $y$ -axis. Find the volume of the solid obtained by revolving  $R$  about the  $y$ -axis.

$$\text{Volume} = \int_0^2 2\pi x(e^{2-x} - 1) dx = 2\pi(e^2 - 5) \approx 15.0109$$

6. For what value of  $k > 0$  do the functions  $f(x) = x^2$  and  $g(x) = 10 - x^2$  have the same average value on the interval  $[0, k]$ ?

$$k = \sqrt{15}$$