Answers to Math 125 Exam #1 Practice Problems

1. $f(x) = 3\sin(\frac{1}{2}x)$

   (a) Estimate the area of the region under the curve of f between $x = 0$ and $x = 2\pi$ using 4 approximating right-end rectangles.

   Area $\approx \frac{\pi}{2}[f(\frac{\pi}{2}) + f(\pi) + f(\frac{3\pi}{2}) + f(2\pi)] \approx 11.3767$

   (b) Write the exact area under the curve of f between $x = 0$ and $x = 2\pi$ as a limit of a sum of approximating right-end rectangles. Do not evaluate this limit.

   Exact Area $= \lim_{n\to\infty} \sum_{i=1}^{n} 3\sin(\frac{\pi i}{n}) \cdot (\frac{2\pi}{n}) \quad (\Delta x = \frac{2\pi}{n},\ x_i = 0 + i\Delta x)$

   (c) Find the exact area under the curve of f between $x = 0$ and $x = 2\pi$ using the definite integrals and the FTC.

   Exact Area $= \int_0^{2\pi} 3\sin(\frac{1}{2}x)\, dx = 12$

2. For $x > 0$, if $g(x) = \int_3^{\ln x} [t^2 + 4\sin(t)]\, dt$, what is $g'(x)$?

   $g'(x) = [(\ln x)^2 + 4\sin(\ln x)] \cdot \frac{1}{x}$

3. If $h(x) = \int_2^4 \frac{e^t}{t^8 - 4t}\, dt$, what is $h'(x)$?

   $h'(x) = 0$ since $h(x)$ is a constant function. It does not vary with x.

4. $\int \frac{x^2}{1 + x^6}\, dx = ?$ (Hint: Choose the right u for substitution.)

   $\int \frac{x^2}{1 + x^6}\, dx = \frac{1}{3}\arctan(x^3) + C \quad (\text{Let } u = x^3.)$

5. $\int_{-\pi/2}^{\pi} [\cos(t) + \frac{3}{\pi}]\, dt = ?$

   $\int_{-\pi/2}^{\pi} [\cos(t) + \frac{3}{\pi}]\, dt = \frac{11}{2}$

6. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = ?$

   $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\sin\sqrt{x} + C \quad (\text{Let } u = \sqrt{x}.)$

7. Is $\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2$ an antiderivative of $x\ln(x)$?

   Yes. Take the derivative of $\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2$.

8. Suppose $f''(x) = x^3 + \frac{1}{x^2}$, $f'(1) = 0$, and $f(1) = 1$. Find $f(x)$.

   $f(x) = \frac{1}{20}x^5 - \ln|x| + \frac{3}{4}x + \frac{1}{5}$