Math 125 Quiz 2 Solutions
April 21, 2006

1. $$\int (3-\sqrt{x})^2\,dx = \int [9-6\sqrt{x}+x]\,dx = 9x - 6x^{\frac{3}{2}}(\frac{2}{3}) + x^2(\frac{1}{2}) + C$$
$$= 9x - 4x^{\frac{3}{2}} + \tfrac{x^2}{2} + C$$

2. $$\int_0^{\pi/4} [sec^2(x) + sec(x)tan(x)]\,dx = tan(x) + sec(x)|_0^{\frac{\pi}{4}}$$
$$= tan(\tfrac{\pi}{4}) + sec(\tfrac{\pi}{4}) - [tan(0) + sec(0)]$$
$$= 1 + \sqrt{2} - [0+1] = \sqrt{2}$$

3. Find $f(x)$ so that $\int_1^x [t^2 f(t) + 3]\,dt = 3x + x^3 + x^4 - 5$. This equality holds if we take the derivative with respect to x on both sides and with the Fund. Thm. of Calc., we have that
$$\tfrac{d}{dx}[\int_1^x [t^2 f(t) + 3]\,dt] = \frac{d}{dx}[3x + x^3 + x^4 - 5]$$
$$x^2 f(x) + 3 = 3 + 3x^2 + 4x^3$$
$$x^2 f(x) = 3x^2 + 4x^3$$
$$f(x) = 3 + 4x$$

4. A European swallow can only fly for 12 hours if carrying a coconut. If the swallow begins flying at $t = 0$, then the airspeed velocity of the laden swallow is given by $v(t) = -\frac{1}{2}t^2 + 4t + 10$ where t is in hours and $v(t)$ is in mph.

   (a) What is the maximum airspeed velocity of the swallow?

   The velocity function is a downward-facing parabola, which means that it has a maximum at its vertex. So the maximum occurs at $t = \frac{-b}{2a} = \frac{-4}{-\frac{1}{2}} = 4$ hours. So the maximum velocity is $v(4) = -\frac{1}{2}4^2 + 4(4) + 10 = 18$ mph.

   (b) What is the distance travelled by the swallow after 12 hours? (Note: The swallow does fly backward for a period of time since it is easily confused.)

   Note that $-\frac{1}{2}t^2 + 4t + 10 = -\frac{1}{2}(t^2 - 8t - 20) = -\frac{1}{2}(t+2)(t-10)$. So, $v(t)$ has roots at $t = -2$ and $t = 10$.

   Thus,

$$|v(t)| = |-\frac{1}{2}t^2 + 4t + 10| = \begin{cases} -\frac{1}{2}t^2 + 4t + 10 & \text{for } -2 \le t \le 10 \\ -(-\frac{1}{2}t^2 + 4t + 10) & \text{for } t < -2 \text{ or } t > 10 \end{cases} \quad (1)$$

$$= \begin{cases} -\frac{1}{2}t^2 + 4t + 10 & \text{for } -2 \leq t \leq 10 \\ \frac{1}{2}t^2 - 4t - 10 & \text{for } t < -2 \text{ or } t > 10 \end{cases} \tag{2}$$

$$\Rightarrow \; Distance\, travelled = \int_0^{12} |v(t)|\, dt = \int_0^{10} -\frac{1}{2}t^2 + 4t + 10\, dt + \int_{10}^{12} \frac{1}{2}t^2 - 4t - 10\, dt$$

$$= [-\tfrac{1}{2}t^3(\tfrac{1}{3}) + 4t^2(\tfrac{1}{2}) + 10t]_0^{10} + [\tfrac{1}{2}t^3(\tfrac{1}{3}) - 4t^2(\tfrac{1}{2}) - 10t]_{10}^{12}$$

$$= [-\tfrac{1}{6}t^3 + 2t^2 + 10t]_0^{10} + [\tfrac{1}{6}t^3 - 2t^2 - 10t]_{10}^{12}$$

$$= [\tfrac{-1000}{6} + 200 + 100] - [0] + [\tfrac{1728}{6} - 288 - 120] - [\tfrac{1000}{6} - 200 - 100]$$

$$= \tfrac{440}{3} = 146\tfrac{2}{3} \text{ miles}$$