Math 125 Quiz 5 Solutions

1. $\int tan^3(x)sec^5(x)\,dx$

   Since the power of $tan(x)$ is odd, reserve $sec(x)tan(x)$ for later and use the identity $sec^2(x) = 1 + tan^2(x)$ to get the rest of the integrand entirely in terms of $sec(x)$.//

$$\begin{aligned}\int tan^3(x)sec^5(x)\,dx &= \int tan^2(x)sec^4(x)(sec(x)tan(x))\,dx \\ &= \int (sec^2-1)sec^4(x)(sec(x)tan(x))\,dx \\ &= \int (u^2-1)u^4\,du = \int u^6 - u^4\,du \\ &= \tfrac{u^7}{7} - \tfrac{u^5}{5} + C = \tfrac{1}{7}sec^7(x) - \tfrac{1}{5}sec^5(x) + C\end{aligned}$$

   (Using the substitution $u = sec(x) \;\Rightarrow\; du = sec(x)tan(x)$)

2. (a) Find the area of the smaller region bounded by the circle $(x-2)^2 + y^2 = 4$ and the line $x = 3$.

   $(x-2)^2 + y^2 = 4$ is a circle with center $(2,0)$ and radius 2. The line $x = 3$ cuts a chord into the circle and we are looking for the area of the chord.

   Solving for y in the circle equation, we get $y = \pm\sqrt{4-(x-2)^2}$ (The plus and minus correspond to the upper and lower semicircles making the circle.)

   Due to the symmetry of the area we are finding, it is enough to find the area of the chord above the $x - axis$ and to double it to get the whole area.
   So total area is described by

$$\text{Area} = 2\int_3^4 \sqrt{4-(x-2)^2}\,dx$$

   Using trigonometric substitution with $x - 2 = 2sin(\theta) \;\Rightarrow\; dx = 2cos(\theta)d\theta$, we have

$$\begin{aligned}\text{Area} &= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{4-4sin^2(\theta)}\cdot 2cos(\theta)\,d\theta = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{4cos^2(\theta)}\cdot 2cos(\theta)\,d\theta \\ &= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4cos^2(\theta)\,d\theta = 8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} cos^2(\theta)\,d\theta \\ &= 8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1+cos(2\theta)}{2}\,d\theta = 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 + cos(2\theta)\,d\theta \\ &= 4[\theta + \tfrac{sin(2\theta)}{2}]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 4[\tfrac{\pi}{2} + \tfrac{sin(\pi)}{2} - (\tfrac{\pi}{6} + \tfrac{sin(\frac{\pi}{3})}{2})] \\ &= 4[\tfrac{\pi}{2} + 0 - \tfrac{\pi}{6} - \tfrac{\sqrt{3}}{4}] = 2.4567\end{aligned}$$

   (b) What is the area of the larger region bounded by the circle $(x-2)^2 + y^2 = 4$ and the line $x = 3$?

   The area of the circle is 4π. So, the area of the larger region is $4\pi - 2.4567 = 10.11$.