Math 125 Antiderivatives Notesheet

Definition: For a function f on [a,b], the function F is an **antiderivative of f** if $F'(x) = f(x)$ for all x in [a,b].

Function	Antiderivative**	Function	Antiderivative**
$c \cdot f(x)$	$c \cdot F(x)$	$sin(x)$	$-cos(x) + C$
$f(x) + g(x)$	$F(x) + G(x)$	$sec^2(x)$	$tan(x) + C$
x^n	$\frac{x^{n+1}}{n+1} + C$	$sec(x)tan(x)$	$sec(x) + C$
$\frac{1}{x}$	$ln\|x\| + C$ (If $x > 0$, $ln(x) + C$)	$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1}(x) + C$
e^x	$e^x + C$	$-\frac{1}{\sqrt{1-x^2}}$	$cos^{-1}(x) + C$
$cos(x)$	$sin(x) + C$	$\frac{1}{1+x^2}$	$tan^{-1}(x) + C$

For C an arbitrary constant

Examples:

1. $f(x) = 2 + 5x + 3x^2 + \frac{1}{x} \quad \Rightarrow \quad F(x) = 2(\frac{x^1}{1}) + 5(\frac{x^2}{2}) + 3(\frac{x^3}{3}) + ln|x| + C$
$= 2x + \frac{5}{2}x^2 + x^3 + ln|x| + C$ for a constant C

2. Find $f(x)$ given that $f''(x) = \frac{x^2+4\sqrt{x}}{x} = x + 4x^{-\frac{1}{2}}$.
By antidifferentiation, we have that $f'(x) = (\frac{x^2}{2}) + 4(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}) + C = \frac{x^2}{2} + 8x^{\frac{1}{2}} + C$.
Again, by antidifferentiation, we have
$f(x) = \frac{1}{2}(\frac{x^3}{3}) + 8(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}) + Cx + D = \frac{x^3}{6} + \frac{16}{3}x^{\frac{3}{2}} + Cx + D$ for constants C and D.

3. Find $f(x)$ if $f'(x) = e^{x+4} - 3$.
Since $\frac{d}{dx}(e^{x+4}) = e^{x+4}$, we have that $f(x) = e^{x+4} - 3x + C$ for a constant C.

4. $f(x) = 4cos(x) + 3sin(x) - sec^2(x)$
$\Rightarrow F(x) = 4sin(x) + 3(-cos(x)) - tan(x) + C = 4sin(x) - 3cos(x) - tan(x) + C$ for a constant C.

NOTE: You can always check your antiderivative by differentiation!!