

## Math 124 Quiz #7 Solutions

1. (a) The velocity is given by  $v = \frac{ds}{dt} = 3t^2 - 18t + 15$ .

The velocity at 4 seconds is  $v = 3(4)^2 - 18(4) + 15 = -9$  feet/sec.

So, the speed is 9 feet/sec.

- (b) The swallow is at rest when  $v = 0 \Rightarrow 3t^2 - 18t + 15 = 0$   
 $\Rightarrow 3(t - 5)(t - 1) = 0$   
 $\Rightarrow t = 1 \text{ or } t = 5 \text{ seconds}$

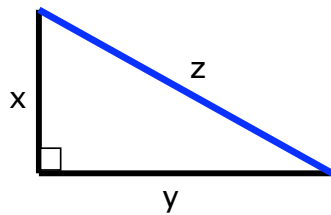
The swallow is moving in the positive direction when  $v > 0$ .

Here are a couple of ways to determine where  $v > 0$ :

- If  $0 \leq t < 1$ , then both of the factors  $t - 1$  and  $t - 5$  are negative, so  $v > 0$ .  
 If  $1 < t < 5$ , then the factor  $t - 1$  is positive, but the factor  $t - 5$  is negative, so  $v < 0$ .  
 If  $t > 5$ , then both of the factors  $t - 1$  and  $t - 5$  are positive, so  $v > 0$ .
- Note that the graph of  $v = 3t^2 - 18t + 15$  is a parabola that opens upward. Since it has zeros at  $t = 1$  and  $t = 5$ , it is positive for  $0 \leq t < 1$  and  $t > 5$ .

So the swallow is at rest at 1 or 5 seconds and it is moving in the positive direction when  $0 \leq t < 1$  and  $t > 5$ .

2. We have the following figure.



Let  $x$  = the distance between the northbound car and the intersection.  
 Let  $y$  = the distance between the eastbound car and the intersection.  
 Let  $z$  = the distance between the cars.

$$\Rightarrow \frac{dx}{dt} = 40 \text{ mph}, \quad \frac{dy}{dt} = 30 \text{ mph}, \quad \text{and} \quad \frac{dz}{dt} = ? \text{ when } x = 1.5 \text{ and } y = 2$$

$$\text{Equation: } x^2 + y^2 = z^2 \quad \Rightarrow \quad 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt} \quad \text{or} \quad x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$$

Plugging in known values to solve for  $\frac{dz}{dt}$ :

Note: When  $x = 1.5$  and  $y = 2$ , we have that  $z = 2.5$ . (Using original equation.)

$$1.5(40) + 2(30) = 2.5 \frac{dz}{dt} \quad \Rightarrow \quad \frac{dz}{dt} = 48 \text{ mph}$$

So, the distance between the cars is increasing by 48 mph when  $x = 1.5$  and  $y = 2$ .