Math 124 Quiz #6 Solutions

1. Using the chain and product rule on the first term and the chain rule on the second term:

$$\frac{d}{dt}[\ln(t\cdot e^t) + 2\arctan(3t)] = \frac{1}{t\cdot e^t}(1\cdot e^t + t\cdot e^t) + \frac{2}{1+(3t)^2}\cdot 3$$

2. Differentiating both sides with respect to x:

$$\frac{d}{dx}(x^2 - \sin y + y^3) = \frac{d}{dx}(1)$$
$$2x - \cos y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

Plugging in $x = 1$ and $y = 0$*:

$$2(1) - \cos(1)\frac{dy}{dx} + 3(0)^2\frac{dy}{dx} = 0$$
$$2 - \frac{dy}{dx} = 0 \qquad \Rightarrow \qquad \frac{dy}{dx} = 2.$$

(*If you solve for $\frac{dy}{dx}$, you should have $\frac{dy}{dx} = \frac{-2x}{-\cos y + 3y^2}$.)

Equation of the tangent line: $y = 2(x-1)$.

3. To get the derivative, I will use logarithmic differentiation because this is not an exponential function (the base is not constant) and I cannot use the power rule (the exponent is not constant).

Taking the natural log of both sides: $\ln y = 3x\cdot\ln x$

Taking the derivative of both sides:

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[3x\cdot\ln x]$$
$$\frac{1}{y}\cdot\frac{dy}{dx} = 3\ln x + \frac{3x}{x}$$
$$\frac{dy}{dx} = y(3\ln x + 3)$$
$$= x^{3x}(3\ln x + 3)$$

So, the slope of of the function is $\left.\frac{dy}{dx}\right|_{x=1} = 1^3(3\ln 1 + 3) = 1(0+3) = 3.$