Math 124 Quiz #4 Solutions

1. (a) Note: $h(x) = 8\sqrt[4]{x} - 2x + 5 = 8x^{1/4} - 2x + 5$

   Power Rule: $$\begin{aligned} h'(x) &= 8(\tfrac{1}{4}x^{1/4-1}) - 2 + 0 \\ &= 2x^{-3/4} - 2 \\ &= \frac{2}{\sqrt[4]{x^3}} - 2 \end{aligned}$$

   (b) Note: We must use the product rule on the term $5t^2e^t$, but we do not need it to take the derivative of the term -1.

   Product Rule: $$\begin{aligned} \frac{dy}{dt} &= 10t \cdot e^t + 5t^2 \cdot e^t - 0 \\ &= 10te^t + 5t^2e^t \end{aligned}$$

2. Note that $g(x) = \frac{1}{x^2} = x^{-2}$.
   The slope of $g(x)$ at any value of x is given by $g'(x) = -2x^{-3} = -\frac{2}{x^3}$.

   We are looking for values of x so that $-\frac{2}{x^3} = -\frac{1}{4}$. $\Rightarrow \quad x^3 = 8$ (Cross-multiplication).
   $\Rightarrow \quad x = 2$.

   So, at $x = 2$, the slope of $g(x)$ is $-\frac{1}{4}$.

3. Note that the power rule reduces the power of a term of the form t^n by 1 with each derivative. So the 52nd derivative of the term $5t^{49}$ is 0. (The powers of t have vanished by the 52nd derivative. In fact, the 50th derivative of $5t^{49}$ is 0.)

   The derivative of $2e^t$ is $2e^t$. After taking one derivative, this term is unchanged. Each additional derivative will leave this term unchanged.

   $$\Rightarrow \quad f^{(52)}(t) = 2e^t.$$