Math 124 Quiz #3 Solutions

1. $f'(8) = 2$: The rate of change of the average height of an 8 year old female is 2 inches per year.
   OR Between the 8th and 9th year, the average female height increases by approximately 2 inches.

   Units: Inches per year

2. (a) • $\lim_{h\to 0} \dfrac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \dfrac{3(x+h)^2+(x+h)-(3x^2+x)}{h}$

$$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2+x+h-3x^2-x}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2+h}{h}$$

$$= \lim_{h\to 0} 6x+3h+1$$

$$= 6x+1$$

• $\lim_{x\to a} \dfrac{g(x)-g(a)}{x-a} = \lim_{x\to a} \dfrac{3x^2+x-(3a^2+a)}{a}$

$$= \lim_{x\to a} \frac{3x^2-3a^2+x-a}{x-a}$$

$$= \lim_{x\to a} \frac{3(x^2-a^2)}{x-a} + \frac{x-a}{x-a}$$

$$= \lim_{x\to a} 3(x+a)+1$$

$$= 6a+1 \qquad \text{So, } g'(x) = 6x+1$$

   (b) The slope of the tangent line at $x = 2$ is given by $m = g'(2) = 6(2)+1 = 13$.

   The point of tangency is $(2, g(2)) = (2, 14)$.
   So, the tangent line is given by $y - 14 = 13(x-2)$ OR $y = 13x - 12$.

3. $h(x)$ has slope 0 at $x = 1$ $\Rightarrow$ $h'(1) = 0$
   For $x < 1$, the slope of $h(x)$ is negative $\Rightarrow$ $h'(x)$ is negative for $x < 1$.
   For $x > 1$, the slope of $h(x)$ is positive $\Rightarrow$ $h'(x)$ is positive for $x > 1$.
   Also, as x increases, $h(x)$ approaches a horizontal asymptote $\Rightarrow$ $h'(x)$ approaches 0 as x increases.