Math 124 Quiz #2 Solutions

1. To find horizontal asymptotes, consider $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$. Since the degree in the numerator and the denominator are the same, both of the limits at infinity will be the ratio of the coefficients of the leading terms, i.e. $\lim_{x\to\infty} \frac{1-x}{3x+6} = \lim_{x\to -\infty} \frac{1-x}{3x+6} = \frac{-1}{3} = -\frac{1}{3}$. $\Rightarrow y = -\frac{1}{3}$ is the only horizontal asymptote.

   To find vertical asymptotes, note that the denominator is $3x+6 = 3(x+2)$. Since the denominator is zero at $x=-2$ and the numerator is **not** zero at $x=-2$, $f(x)$ has the vertical asymptote $x=-2$.

2. (a) Here are two ways to approach the problem:

   - $$\begin{aligned}\lim_{x\to -\infty} \frac{-x^5+3}{25x^4+10x^2+8} &= \lim_{x\to -\infty} \frac{-x^5+3}{25x^4+10x^2+8}\cdot\frac{1/x^4}{1/x^4} \\ &= \lim_{x\to -\infty} \frac{-x+\frac{3}{x^4}}{25+\frac{10}{x^2}+\frac{8}{x^4}} \\ &= \infty \qquad \text{since the terms } \tfrac{3}{x^4},\ \tfrac{10}{x^2},\ \tfrac{8}{x^4} \text{ approach 0 as } x\to-\infty \\ &\qquad\quad \text{and } -x\to\infty \text{ as } x\to-\infty\end{aligned}$$

   - Since the degree of the numerator is greater than the degree of the denominator, $\lim_{x\to -\infty} \frac{-x^5+3}{25x^4-10x^2+8}$ is either $+\infty$ or $-\infty$.

     Note that $-x^5$ is positive when x is negative. So, as $x\to-\infty$, $-x^5+3$ will be positive.
     Also note that $25x^4$ is positive when x is negative. So, as $x\to-\infty$, $25x^4-10x^2+8$ will be positive ($25x^4$ is the dominant term).
     Since both the numerator and denominator are positive as $x\to-\infty$, we must have that
     $$\lim_{x\to -\infty} \frac{-x^5+3}{25x^4+10x^2+8} = \infty.$$

   (b) Since the function $\frac{2}{\sqrt{\sin(\pi t)+t+2}}$ is defined at $t=3$, the limit can be evaluated by evaluating the function at $t=3$.

   $$\Rightarrow \quad \lim_{t\to 3} \frac{2}{\sqrt{\sin(\pi t)+t+2}} = \frac{2}{\sqrt{\sin(3\pi)+3+2}} = \frac{2}{\sqrt{5}}.$$

3. Note that $\frac{1}{x-1}$ is discontinuous $x=1$ and continuous for all other values of $x>0$, so $g(x)$ is continuous for $x>0$ except at $x=1$.

   Also note that x^2-1 is continuous for all values $x<0$.

   The only other x-value that must be checked for continuity is $x=0$. Note that:

   - $g(0) = 0^2-1 = -1$.
   - $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} x^2-1 = -1$
   - $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \frac{1}{x-1} = -1$

   $$\Rightarrow \quad \lim_{x\to 0} g(x) = -1$$

   Since $g(0) = \lim_{x\to 0} g(x)$, g is continuous at $x=0$. (The function does **not** have a jump at $x=0$.)

   So, the function $g(x)$ is continuous at all real values except $x=1$ or $(-\infty,1)\cup(1,\infty)$.