Math 124
Final Exam
June 11th, 2008

Name: ______________________________

1. Your exam contains 8 questions and 8 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 2 hours for this exam.
3. Make sure to **ALWAYS SHOW YOUR WORK**; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. **Note:** To evaluate limits, proof by graph or table of values does not suffice for full credit.
4. If you need extra space, attach an extra sheet of paper to the back of the exam and clearly indicate this.
5. You are allowed two 8.5×11 sheets of handwritten notes (both sides) and a graphing or scientific calculator.
6. Leave answers in exact form (as simplified as possible).
7. Put a $\boxed{\text{box around your final answer}}$ where applicable.

Here are some handy formulas:
Areas Circle: $A = \pi r^2$ Triangle: $A = \frac{1}{2}bh$ Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$

Volumes Sphere: $V = \frac{4}{3}\pi r^3$ Cone: $V = \frac{1}{3}\pi r^2 h$ Cylinder: $V = \pi r^2 h$

Problem	Total Points	Score
1	16	
2	7	
3	11	
4	14	
5	11	
6	15	
7	12	
8	14	
Total	100	

1. (16 pts.) Find the following. **Do not simplify** your answers.

   (a) (8 pts.) Find $f'(x)$ if $f(x) = (\ln x \cdot e^{\tan x})^{12} + \sqrt{2} \cdot x^3$.

   (b) (8 pts.) $\frac{d}{dt}\left[\frac{\sqrt{\arctan(5t)}}{6t^2+8} - 3t\right] = ?$

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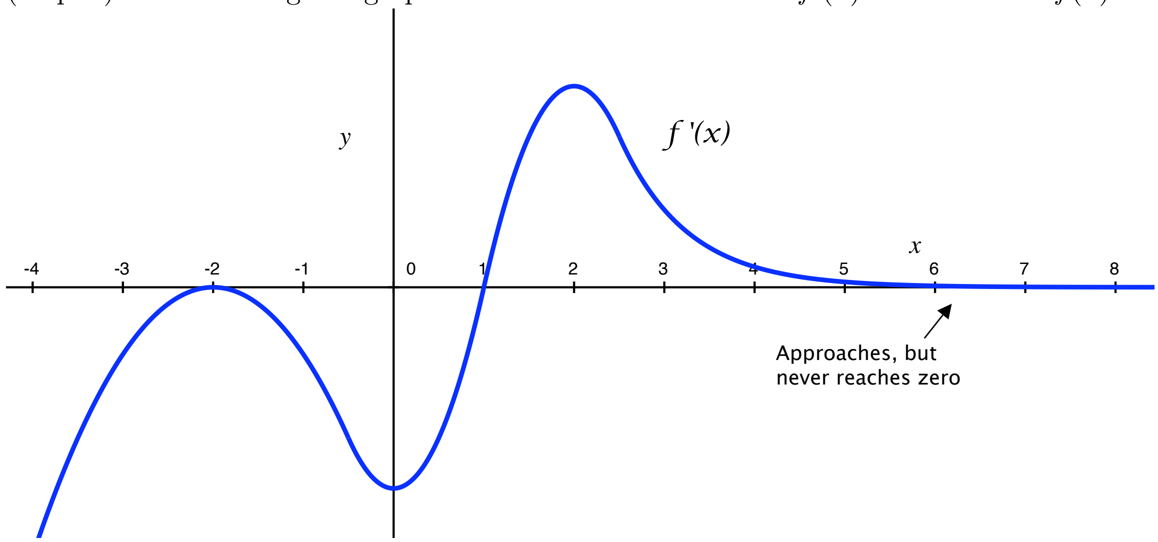
2. (7 pts.) Suppose the height of the tide (in feet) is given by the function $h(t)$ at t **hours past noon**.

   If the height of tide at **2 pm is 7 feet** and $h'(2) = -3$ feet/hour, approximate the height of the tide at 2:15 pm using linear approximation.

3. (11 pts.) The following is a graph of the **derivative** function $f'(x)$ for a function $f(x)$.



Answer the following questions about the **original** function $f(x)$.

(a) (3 pts.) On what intervals is the $f(x)$ decreasing?

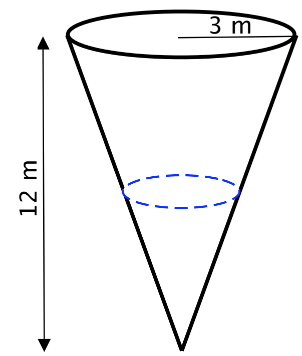
(b) (4 pts.) For what x-values (if any) does $f(x)$ have a local maximum?

For what x-values (if any) does $f(x)$ have a local minimum?

(c) (4 pts.) On what intervals is $f(x)$ concave up? On what intervals is $f(x)$ concave down?

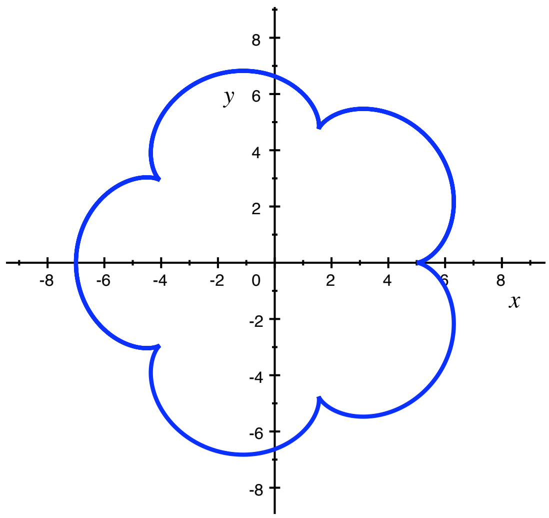
4. (14 pts.) Water is being pumped into an inverted conical tank at a rate of 2 m^3/min. The tank has a height of 12 meters and a radius of 3 meters at the top.

(a) (11 pts.) At what **rate** is the water level rising when the water is 4 meters deep? Include **units** in your answer.



(b) (3 pts.) How long does it take to fill up the tank?

5. (11 pts.) Consider the following parametric curve (a ranunculoid) described by the equations below.



$$x(t) = 6\cos t - \cos(6t)$$

$$y(t) = 6\sin t - \sin(6t)$$

Find the **equation** of the tangent line at $t = \dfrac{\pi}{2}$.

6. (15 pts.) $f(x) = 2x^3 \cdot e^x$

(a) (10 pts.) Find the critical numbers of $f(x)$ and identify each number as the location of a local maximum, local minimum, or neither by using the 1st or 2nd derivative tests.

(b) (5 pts.) If the domain of $f(x)$ is restricted to $[-4, 1]$, what are the **global** maximum and minimum **values** of $f(x)$ on this domain?

7. (12 pts.) Evaluate the following limits. **Justify** your answers. If the limit is infinite, determine if it is $+\infty$ or $-\infty$.

(a) (4 pts.) $\lim_{x \to 1^+} \frac{4|x-1|}{x^2-1}$

(b) (4 pts.) $\lim_{x \to 1} \frac{3x-3}{\ln x}$

(c) (4 pts.) $\lim_{t \to 2^-} \frac{e^t}{(t-2)^3}$

8. (14 pts.) $f(x) = \frac{x^2-16}{3x^2+12x} + 2e^x$

(a) (6 pts.) For what x-values is $f(x)$ continuous?

(b) (6 pts.) Evaluate the following limits. **Justify** your answers. If the limit is infinite, determine if it is $+\infty$ or $-\infty$.

i. $\lim_{x\to\infty} f(x)$

ii. $\lim_{x\to-\infty} f(x)$

(c) (2 pts.) Find **equations** of the horizontal and vertical asymptotes of $f(x)$ (if any exist).