Math 124
Exam 2 Solutions

1. (a) Using the product and chain rules:
$\frac{d}{dt}[\sec(\pi t)\cdot(e^t+5)] = \sec(\pi t)\cdot\tan(\pi t)\cdot\pi\cdot(e^t+5) \ + \ \sec(\pi t)\cdot e^t$

(b) Using the chain rule twice:
$f'(x) = \frac{1}{2}(\ln(3x^2-7))^{-1/2}\cdot\frac{1}{3x^2-7}\cdot 6x$

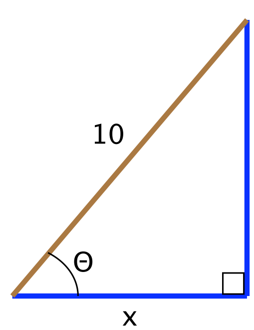
(c) Using the quotient and chain rules:
$$\frac{d}{dx}\left[\frac{x^3-1}{\arcsin(2x)}\right] = \frac{\arcsin(2x)\cdot 3x^2-(x^3-1)\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2}{(\arcsin(2x))^2}$$

(d) Using logarithmic differentiation:
$$\begin{aligned} \ln y &= t\cdot\ln(\sin t) \\ \Rightarrow \quad \frac{1}{y}\frac{dy}{dx} &= 1\cdot\ln(\sin t)+t\cdot\frac{1}{\sin t}\cdot\cos t \\ \frac{dy}{dx} &= (\sin t)^t[\ln(\sin t)+t\cdot\cot t] \end{aligned}$$

2. Note that $g'(x) = e^{-2x}(-2)+0 = -2e^{-2x}$,
$g''(x) = -2e^{-2x}(-2) = 4e^{-2x}$,
$g'''(x) = 4e^{-2x}(-2) = -8^{-2x}$, and so on.

Following the pattern, we will have that $g^{(23)} = (-2)^{23}e^{-2x} = -2^{23}e^{-2x}$.

3. Let x = the distance between the base of the ladder and the wall.



$\frac{dx}{dt} = 5$ ft/sec when $x=5$

$\frac{d\theta}{dt} = \ ?$ when $x=5$

We can relate θ and x with the following equation: $\cos\theta = \frac{x}{10}$

Differentiating both sides: $-\sin\theta\cdot\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{dx}{dt}$

Note that when $x=5$, $\frac{dx}{dt} = 5$, and $\cos\theta = \frac{5}{10} = \frac{1}{2}$.

Using a reference triangle of the Pythagorean identity, we have that when $x = 5$, $\sin\theta = \frac{\sqrt{3}}{2}$.

Plugging in known values: $-\frac{\sqrt{3}}{2} \cdot \frac{d\theta}{dt} = \frac{1}{10}(5) \quad \Rightarrow \quad \frac{d\theta}{dt} = -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$ radians/sec.

4. (a) The squirrel is at rest when the velocity is equal to zero. $\Rightarrow \quad t = 1,\ t = 3,\ t = 5$ seconds

(b) The squirrel is moving forward when the velocity is positive. $\Rightarrow \quad 0 \le t < 1,\ 3 < t < 5,\ t > 5$

(c) The acceleration is positive when the slope of the velocity is positive. $\Rightarrow \quad 2 < t < 3.75,\ t > 5$

(d) The squirrel is speeding up when the speed $= |\text{velocity}|$ is increasing.
$\Rightarrow \quad 1 < t < 2,\ 3 < t < 3.75,\ t > 5$

5. (a) The point $(2, 0)$ is on the curve if the values $x = 2$ and $y = 0$ satisfy the given equation.

Left-hand side evaluated at $x = 2$, $y = 0$: $e^0 + 2^2(0) = 1$
Right-hand side evaluated at $x = 2$, $y = 0$: $\frac{1}{2}(2) + 0^3 = 1$

Since these are equal, the point $(2, 0)$ is on the curve.

(b) To find the equation of the tangent line, we must find the slope of the curve at $(2, 0)$ or $\left.\frac{dy}{dx}\right|_{x=2,\ y=0}$.

Differentiating both sides of the equation with respect to x:

$$e^y\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} = \frac{1}{2} + 3y^2\frac{dy}{dx}$$

Plugging in the values $x = 2$ and $y = 0$:

$$e^0\frac{dy}{dx} + 2(2)(0) + 2^2\frac{dy}{dx} = \frac{1}{2} + 3(0)^2\frac{dy}{dx}$$

$$\frac{dy}{dx} + 4\frac{dy}{dx} = \frac{1}{2}$$

$$5\frac{dy}{dx} = \frac{1}{2} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{10}$$

Equation of the Line: $y = \frac{1}{10}(x - 2)$