Math 124
Exam 1 Solutions

1. (a) $\lim_{x\to 5} \frac{6x-30}{x^2-7x+10} = \lim_{x\to 5} \frac{6(x-5)}{(x-5)(x-2)} = \lim_{x\to 5} \frac{6}{x-2} = \frac{6}{5-2} = 2$

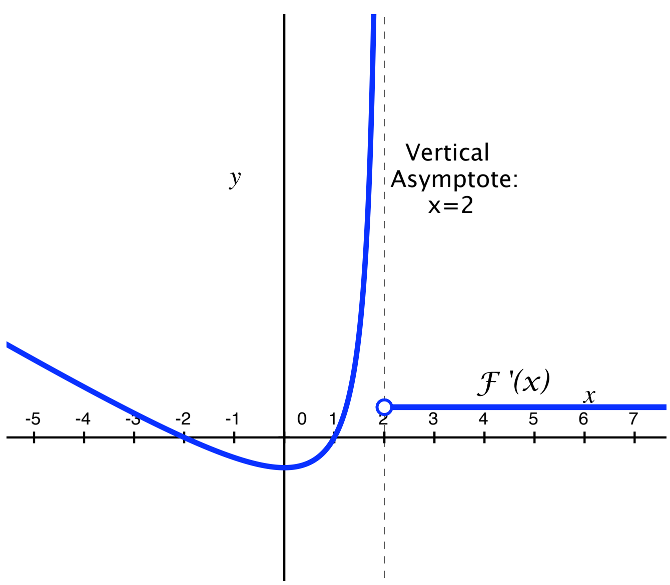
   (b) $\lim_{x\to 2^-} \frac{6x-30}{x^2-7x+10} = \lim_{x\to 2^-} \frac{6}{x-2} = -\infty$ since the numerator is 6 (positive) and the denominator is negative and approaching zero. (Since $x<2$, $x-2<0$.)

   (c) $\lim_{t\to -\infty} \frac{9t-\frac{3}{t}}{-4t+8} = -\frac{9}{4}$ since the highest power in the numerator and the denominator match. ($\frac{3}{t}\to 0$ as $t\to -\infty$)

   (d) $\lim_{t\to \pi/2} \frac{\cos^2 t - 3}{\sin t} = \frac{\cos^2(\frac{\pi}{2})-3}{\sin(\frac{\pi}{2})} = \frac{0-3}{1} = -3$

2. $F(x)$ has slope zero at $x=-2$ and $x=1$ (approximately). $\Rightarrow$ $F'(x)=0$ at $x=-2$ and $x=1$.

   For $x<-2$, the slope of $F(x)$ is positive $\Rightarrow$ $F'(x)$ is positive for $x<-2$.
   For $-2<x<1$, the slope of $F(x)$ is negative. $\Rightarrow$ $F'(x)$ is negative for $-2<x<1$.
   For $1<x<2$, the slope of $F(x)$ is positive and increasing without bound as $x\to 2$.
   $\Rightarrow$ $F'(x)$ is positive and increasing without bound as $x\to 2$.
   For $x>2$, the slope of $F(x)$ is constant and positive. $\Rightarrow$ $F'(x)$ is positive for $x>2$.



3. (a) To investigate the discontinuities of $f(x)$, we can look at the discontinuities of each term.

   Note that $4\arctan x$ is continuous for all real values since it is defined for all real values.

Also note that $\frac{5x^3}{x^5-x^3} = \frac{5x^3}{x^3(x^2-1)}$ is undefined for $x=0$, $x=1$, and $x=-1$ and defined elsewhere.

So, $\frac{5x^3}{x^5-x^3}$ is discontinuous at $x=0$, $x=1$, and $x=-1$.

Since $\frac{5x^3}{x^5-x^3} = \frac{5}{(x+1)(x-1)}$ for $x \neq 0$, there is a hole at $x=0$ and vertical asymptotes at $x=1$ and $x=-1$.

$\Rightarrow x=0$: Removable discontinuity $\qquad$ $x=1$ and $x=-1$: Infinite discontinuities

(b) As stated in part (a), the vertical asymptotes of $f(x)$ are $x=1$ and $x=-1$.

To find horizontal asymptotes, note that

$$\lim_{x\to\infty} \frac{5x^3}{x^5-x^3} + 4\arctan x = 0 + 4(\frac{\pi}{2}) = 2\pi$$

and

$$\lim_{x\to-\infty} \frac{5x^3}{x^5-x^3} + 4\arctan x = 0 + 4(-\frac{\pi}{2}) = -2\pi$$

So, there are two horizontal asymptotes: $y = 2\pi$ and $y=-2\pi$

4. The velocity at $t=4$ will be given by $\lim_{t\to4} \frac{H(t)-H(4)}{t-4}$ or $\lim_{h\to0} \frac{H(4+h)-H(4)}{h}$ if these limits exist.

Note that $H(4) = -16(4)^2 + 520 = 264$.

- $$\begin{aligned} \lim_{t\to4} \frac{H(t)-H(4)}{t-4} = \lim_{t\to4} \frac{-16t^2+520-264}{t-4} &= \lim_{t\to4} \frac{-16t^2+256}{t-4} \\ &= \lim_{t\to4} \frac{-16(t+4)(t-4)}{t-4} \\ &= \lim_{t\to4} -16(t+4) \\ &= -128 \text{ ft/sec} \end{aligned}$$

- $$\begin{aligned} \lim_{h\to0} \frac{H(4+h)-H(4)}{h} &= \lim_{h\to0} \frac{-16(4+h)^2+520-264}{h} \\ &= \lim_{h\to0} \frac{-16(16+8h+h^2)+256}{h} \\ &= \lim_{h\to0} \frac{-128h-16h^2}{h} \\ &= \lim_{h\to0} -128-16h \\ &= -128 \text{ ft/sec} \end{aligned}$$

5. (a)

- $$\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = \lim_{h\to 0}\frac{5(x+h)+\sqrt{3(x+h)+1}-(5x-\sqrt{3x+1})}{h}$$
$$= \lim_{h\to 0}\frac{5h+\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}$$
$$= \lim_{h\to 0}\frac{5h}{h}+\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}$$
$$= \lim_{h\to 0} 5+\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}\cdot\frac{\sqrt{3x+3h+1}+\sqrt{3x+1}}{\sqrt{3x+3h+1}+\sqrt{3x+1}}$$
$$= \lim_{h\to 0} 5+\frac{3h}{h(\sqrt{3x+3h+1}+\sqrt{3x+1})}$$
$$= \lim_{h\to 0} 5+\frac{3}{\sqrt{3x+3h+1}+\sqrt{3x+1}}$$
$$= 5+\frac{3}{2\sqrt{3x+1}}$$

- $$\lim_{x\to a}\frac{g(x)-g(a)}{x-a} = \lim_{x\to a}\frac{5x+\sqrt{3x+1}-(5a-\sqrt{3a+1})}{x-a}$$
$$= \lim_{x\to a}\frac{5x-5a}{x-a}+\frac{\sqrt{3x+1}-\sqrt{3a+1}}{x-a}$$
$$= \lim_{x\to a} 5+\frac{\sqrt{3x+1}-\sqrt{3a+1}}{x-a}\cdot\frac{\sqrt{3x+1}+\sqrt{3a+1}}{\sqrt{3x+1}+\sqrt{3a+1}}$$
$$= \lim_{x\to a} 5+\frac{3x-3a}{(x-a)(\sqrt{3x+1}+\sqrt{3a+1})}$$
$$= \lim_{x\to a} 5+\frac{3}{\sqrt{3x+1}+\sqrt{3a+1}}$$
$$= 5+\frac{3}{2\sqrt{3a+1}}$$

So, $g'(x) = 5+\frac{3}{2\sqrt{3x+1}}$.

(b) The slope of the tangent line is given by $g'(0) = 5+\frac{3}{2} = \frac{13}{2}$.

The point of tangency is $(0, g(0)) = (0, 0+\sqrt{1}) = (0, 1)$.

So, the equation is $y = \frac{13}{2}x + 1$.