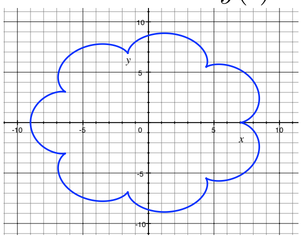
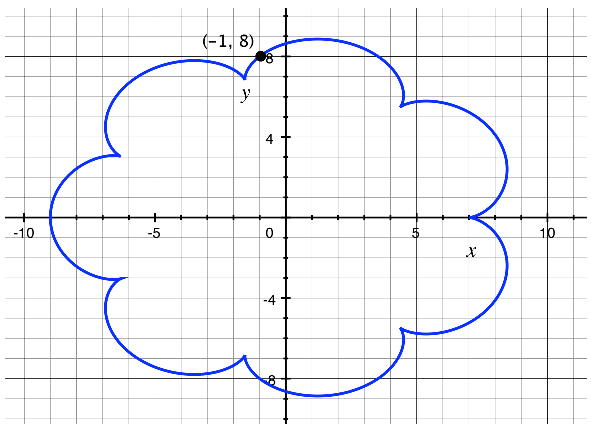
Math 124 Worksheet #6

1. The equations $x(t) = 8\cos(t) - \cos(8t)$ $(0 \le t \le 2\pi)$ trace out an epicycloid.
$y(t) = 8\sin(t) - \sin(8t)$



(a) Indicate the approximate location on the graph of the point at which $t = \frac{\pi}{2}$.



Plugging $t = \frac{\pi}{2}$ into the parametric equations:

$x = 8\cos(\frac{\pi}{2}) - \cos(8 \cdot \frac{\pi}{2}) = -1$
$y = 8\sin(\frac{\pi}{2}) - \sin(8 \cdot \frac{\pi}{2}) = 8$

$\rightarrow$ Point: $(-1, 8)$

(b) Find the equation of the tangent line when $t = \frac{\pi}{2}$.

We have the point $(-1, 8)$ at which the line is tangent to the curve. We now need the slope of the line.

$\frac{dx}{dt} = -8\sin(t) + 8\sin(8t)$ and $\frac{dy}{dt} = 8\cos(t) - 8\cos(8t)$

So, $\frac{dy}{dx} = \frac{8\cos(t) - 8\cos(8t)}{-8\sin(t) + 8\sin(8t)}$ and $\frac{dy}{dx}\Big|_{t=\pi/2} = \frac{8\cos(\frac{\pi}{2}) - 8\cos(8\cdot\frac{\pi}{2})}{-8\sin(\frac{\pi}{2}) + 8\sin(8\cdot\frac{\pi}{2})} = 1$

Equation of Tangent Line: $y - 8 = 1(x + 1)$ or $y = x + 9$

2. Find the extreme values of the following functions on each indicated interval.

(a) $f(x) = x^3 - 3x + 1$ on $[-3, 2]$

i. **Finding critical numbers:**

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1)$$

The derivative is defined for all x-values so to find critical numbers, we have to find where it is equal to zero. $\Rightarrow$ $3(x + 1)(x - 1) = 0$

So, our critical numbers in the domain are $x = -1$ and $x = 1$.

ii. **Evaluating f at the endpoints and critical numbers:**

$f(-3) = -17$ $\quad$ $f(-1) = 3$
$f(2) = 3$ $\quad$ $f(1) = -1$

iii. **Stating max and min values:**

The absolute maximum value is 3 and it occurs at $x = -1$ and $x = 2$.
The absolute minimum value is -17 and it occurs at $x = -3$.

(b) $g(x) = \frac{3x^2}{x-3}$ on $[4, 8]$

i. **Finding critical numbers:**

$$g'(x) = \frac{(x-3)(6x)-3x^2(1)}{(x-3)^2} = \frac{3x^2-18x}{(x-3)^2}$$

The derivative is undefined for $x = 3$. However, $x = 3$ is not in the domain, so it is not a critical number. To find where $g'(x) = 0$, we must find where $3x^2 - 18x = 0 \quad \Rightarrow \quad 3x(x-6) = 0 \quad \Rightarrow \quad x = 0$ or $x = 6$

Since $x = 0$ is also not in the domain, we only have one critical number $x = 6$.

ii. **Evaluating g at the endpoints and critical numbers:**

$g(4) = 48$ $\quad$ $g(6) = 36$
$g(8) = 38.4$

iii. **Stating max and min values:**

The absolute maximum value is 48 and it occurs at $x = 4$.
The absolute minimum value is 36 and it occurs at $x = 6$.

(c) $h(x) = x^2e^{-4x}$ on $[-.1, 4]$

i. **Finding critical numbers:**

$$h'(x) = 2xe^{-4x} + x^2(-4e^{-4x}) = 2xe^{-4x}(1 - 2x)$$

The derivative is defined for all x-values so to find critical numbers, we have to find where it is equal to zero. $\quad \Rightarrow \quad 2xe^{-4x}(1 - 2x) = 0$

Since $e^{-4x} > 0$ for all x-values, we have that our critical numbers in the domain are $x = 0$ and $x = \frac{1}{2}$.

ii. **Evaluating f at the endpoints and critical numbers:**

$h(-.1) \approx .0067$ $\quad$ $h(0) = 0$
$h(4) \approx 1.8 \times 10^{-6}$ $\quad$ $h(\frac{1}{2}) \approx 0.33834$

iii. **Stating max and min values:**

The absolute maximum value is approximately 0.33834 and it occurs at $x = \frac{1}{2}$.
The absolute minimum value is 0 and it occurs at $x = 0$.

3. $f(x) = 1 - 3xe^{\frac{1}{4}x}$

(a) Find the linearization of f at $x = 0$.

$$f'(x) = -3e^{\frac{1}{4}x} - 3xe^{\frac{1}{4}x} \cdot \frac{1}{4} \quad \Rightarrow \quad f'(0) = -3$$

So the tangent line (linearization) has slope -3.

$f(0) = 1 \quad \Rightarrow \quad$ The tangent line (linearization) goes through the point $(0, 1)$.

$$\text{Linearization: } L(x) = f(0) + f'(0)(x - 0) = 1 - 3x$$

(b) Use the linearization to approximate the x-value for which $f(x) = 2$. (Trying to algebraically solve for x such that $f(x) = 2$ is difficult, so this is a good time for approximation.)

If $f(x) = 2$, then $L(x) \approx 2 \quad \Rightarrow \quad 2 = 1 - 3x$

$$x = -\frac{1}{3}$$

So, $f(x) = 2$ for $x \approx -\frac{1}{3}$.