Math 124 Worksheet #3 Solutions

1. The following limit represents the derivative of some function $f(x)$ at some number a. What is $f(x)$ and a?

$$\lim_{h\to 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h}$$

This limit represents the derivative of the function $f(x) = \dfrac{1}{x^2}$ at $a = 2$.

2. Suppose $v = g(t)$ gives the velocity (in meters/second) of a(n) ________________ at
object
t seconds as it travels along a straight line.

(a) What is the meaning of the derivative $g'(t) = \dfrac{dv}{dt}$? What are the units of the derivative?

I am choosing my object to be a penguin. So, the derivative $g'(t)$ gives the rate of change of the velocity of penguin with respect to time at t seconds. Note that the rate of change of velocity with respect to time is also known as the acceleration, so $g'(t)$ gives the acceleration of the penguin at t seconds. The units of the derivative will be the units of $g(t)$ divided by the units of t $\rightarrow$ $\dfrac{m/s}{s} = \dfrac{m}{s^2}$

(b) Interpret the statement $\dfrac{dv}{dt}\Big|_{t=10} = 6$.

This statement tells us that the acceleration of the penguin at 10 seconds is 6 meters/second^2.

3. For $y = \dfrac{1}{\sqrt{3x}}$, find $\dfrac{dy}{dx}$ using limits.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{3(x+h)}} - \frac{1}{\sqrt{3x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{3x}}{\sqrt{3x}\sqrt{3x+3h})} - \frac{\sqrt{3x+3h}}{\sqrt{3x}\sqrt{3x+3h}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{3x}-\sqrt{3x+3h}}{\sqrt{3x}\sqrt{3x+3h}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{3x}-\sqrt{3x+3h}}{h\sqrt{3x}\sqrt{3x+3h}}$$

$$= \lim_{h\to 0} \frac{\sqrt{3x}-\sqrt{3x+3h}}{h\sqrt{3x}\sqrt{3x+3h}} \cdot \frac{\sqrt{3x}+\sqrt{3x+3h}}{\sqrt{3x}+\sqrt{3x+3h}} \quad (\times \text{ by conjugate})$$

$$= \lim_{h \to 0} \frac{3x-(3x+3h)}{h\sqrt{3x}\sqrt{3x+3h}(\sqrt{3x}+\sqrt{3x+3h})}$$

$$= \lim_{h \to 0} \frac{-3h}{h\sqrt{3x}\sqrt{3x+3h}(\sqrt{3x}+\sqrt{3x+3h})}$$

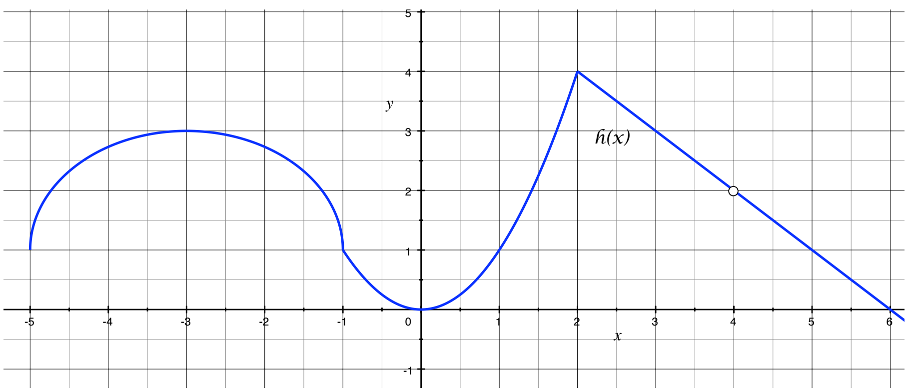
$$= \lim_{h \to 0} \frac{-3}{\sqrt{3x}\sqrt{3x+3h}(\sqrt{3x}+\sqrt{3x+3h})}$$

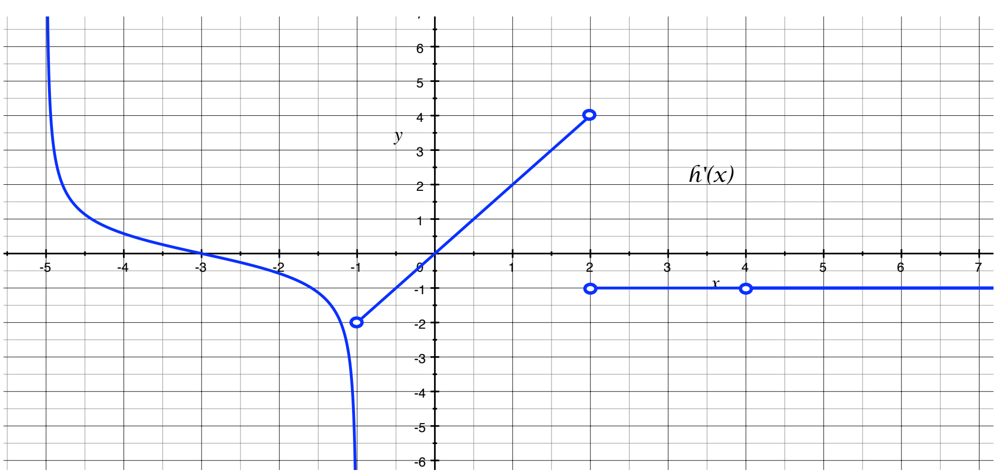
$$= \frac{-3}{\sqrt{3x}\sqrt{3x+3(0)}(\sqrt{3x}+\sqrt{3x+3(0)})} \quad \text{(Plugging in } h=0\text{)}$$

$$= \frac{-3}{3x(2\sqrt{3x})}$$

$$= \frac{-1}{2x\sqrt{3x}} \text{ for } x > 0$$

4. The function $h(x)$ is given below. For what values of x is $h(x)$ differentiable? Sketch $h'(x)$.





The function h is differentiable for the on the following intervals: $(-5,-1)$, $(-1,2)$, $(2,4)$, $(4,\infty)$