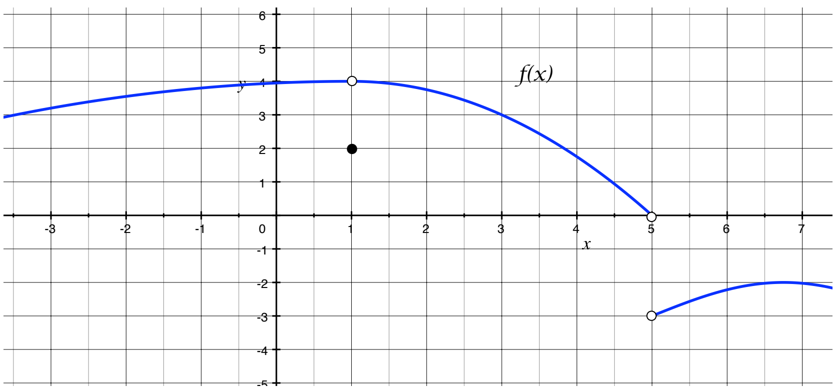
Math 124 Worksheet #1 Solutions

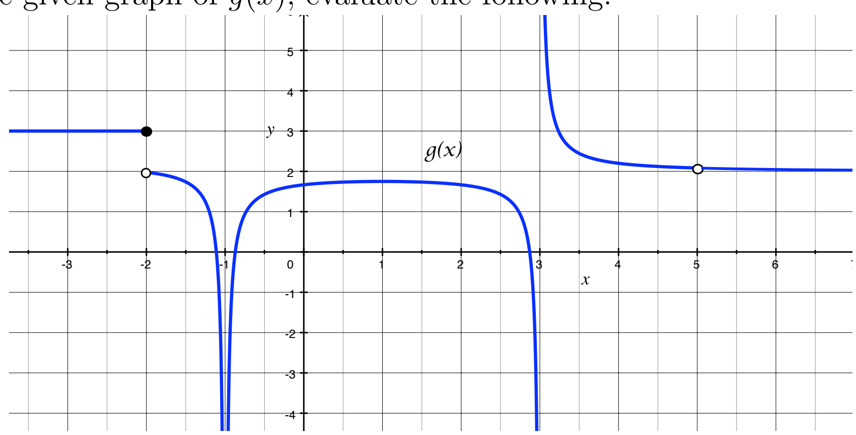
1. Sketch a possible graph of a function f that satisfies the following conditions.

$$f(1) = 2, \quad \lim_{x \to 1} f(x) = 4, \quad \lim_{x \to 5^-} f(x) = 0, \quad \lim_{x \to 5^+} f(x) = -3$$

Here is a possible graph that satisfies the conditions.



2. For the given graph of $g(x)$, evaluate the following.



$\lim_{x \to -2^-} g(x)$ $\lim_{x \to -2^+} g(x)$ $\lim_{x \to -2} g(x)$

$\lim_{x \to -1} g(x)$ $\lim_{x \to 3^-} g(x)$ $\lim_{x \to 3^+} g(x)$

$\lim_{x \to 5} g(x)$

- Looking at the graph as x approaches -2 from the left, we see that the function $g(x)$ is approaching the y-value 3. So, $\lim_{x \to -2^-} g(x) = 3$. Similarly, as x approaches -2 from the right, $g(x)$ approaches the value 2. So, $\lim_{x \to -2^+} g(x) = 2$. Since the left-hand and right-hand limits are not equal as $x \to -2$, $\lim_{x \to -2} g(x)$ does not exist.

- As x approaches -1 (from the left or right), the function $g(x)$ decreases without bound. So, $\lim\limits_{x\to -1} g(x) = -\infty$.

- As x approaches 3 from the left, the function $g(x)$ decreases without bound. So, $\lim\limits_{x\to 3^-} g(x) = -\infty$. As x approaches 3 from the right, $g(x)$ increases without bound. So $\lim\limits_{x\to 3^+} g(x) = \infty$.

- As x approaches 5 (from the left or right), $g(x)$ approaches the y-value 2 (approximately). So, $\lim\limits_{x\to 5} g(x) = 2$.

3. Evaluate the following.

   (a) $\lim\limits_{x\to\sqrt{2}} 2x^6 - 3$

   Since $2x^6 - 3$ is a polynomial, we can evaluate the limit by plugging the value $x = \sqrt{2}$ directly into the expression. $\Rightarrow \lim\limits_{x\to\sqrt{2}} 2x^6 - 3 = 2(\sqrt{2})^6 - 3 = 2(8) - 3 = 13.$

   (b) $\lim\limits_{t\to 1} (t^2+1)(3t^3-4t+2)$

   Recall: The limit of a product if the product of the limits, provided that each limit exists.
   So, $\lim\limits_{t\to 1} (t^2+1)(3t^3-4t+2) = \lim\limits_{t\to 1} (t^2+1) \cdot \lim\limits_{t\to 1} (3t^3-4t+2) = 2 \cdot 1 = 2.$

   (c) $\lim\limits_{x\to -1} \frac{x-3}{3x^2+3}$

   Since the rational expression is defined at $x = -1$, we can evaluate the limit by plugging the value $x = -1$ directly into the expression.
   So, $\lim\limits_{x\to -1} \frac{x-3}{3x^2+3} = \frac{-1-3}{3(-1)^2+3} = \frac{-4}{6} = -\frac{2}{3}.$

   (d) $\lim\limits_{t\to 0} \frac{3t^2+6t}{t^3-4t}$

   We cannot plug $t = 0$ directly into the expression to evaluate the limit since the expression is undefined for that value.

   Note that $\frac{3t^2+6t}{t^3-4t} = \frac{t(3t+6)}{t(t^2-4)} = \frac{3t+6}{t^2-4}$ for $t \neq 0$, $t \neq 2$, or $t \neq -2$.

   So, $\lim\limits_{t\to 0} \frac{3t^2+6t}{t^3-4t} = \lim\limits_{t\to 0} \frac{3t+6}{t^2-4} = \frac{3(0)+6}{0^2-4} = \frac{6}{-4} = -\frac{3}{2}.$