Calculus I - Math 124 Exam #2 Review Answers

- Which Rule?
  1. $\frac{d}{dx}\left[\frac{x}{\sin\frac{\pi}{4}}\right] = \sqrt{2}$ (Constant multiple of x.)
  2. $\frac{d}{dx}[\pi\sin(x^4)] = 4\pi x^3\cos(x^4)$ (Chain Rule and Power Rule)
  3. $\frac{d}{dx}[(\sin\ x)^{\pi/4}] = \frac{\pi}{4}(\sin\ x)^{\frac{\pi}{4}-1}\cos\ x$ (Chain Rule and Power Rule)
  4. $\frac{d}{dx}[(\sin\frac{\pi}{4})^x] = (\sin\frac{\pi}{4})^x\ln(\sin\frac{\pi}{4})$ (Exponential Function with base $\sin\frac{\pi}{4}$)
     or $(\frac{1}{\sqrt{2}})^x\ln(\frac{1}{\sqrt{2}})$
  5. $\frac{d}{dx}[x^{\sin\ x}] = x^{\sin\ x}(\cos\ x\ \ln\ x + \frac{\sin\ x}{x})$ (Logarithmic differentiation)

- Chain Rule!!!
  1. $\frac{d}{dr}[\cos\ 2^r] = (-\sin\ 2^r)\cdot 2^r\ln\ 2$
  2. $g(x) = \sqrt{5}^{3x^2+x} \quad\rightarrow\quad g'(x) = \sqrt{5}^{3x^2+x}\ln\sqrt{5}\cdot(6x+1)$
  3. $\frac{d}{d\theta}[\sec(\tan\ \theta) + 5\theta] = \sec(\tan\ \theta)\tan(\tan\ \theta)\cdot\sec^2\theta + 5$
  4. $h(x) = \frac{e^{-1/x}}{2x} + 1 \quad\rightarrow\quad h'(x) = \frac{2x\cdot e^{-1/x}\cdot\frac{1}{x^2} - e^{-1/x}\cdot 2}{4x^2}$
  5. $\frac{d}{dx}[3(\arcsin\ 2x)^5] = 3\cdot 5(\arcsin\ 2x)^4\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2$

- Fun with logs
  1. $\frac{d}{d\theta}[\ln(\sin\ \theta)] = \frac{1}{\sin\ \theta}\cdot\cos\ \theta = \cot\ \theta$
  2. $\frac{d}{dz}[2\ln(1 + e^z + \sqrt[4]{z})] = \frac{2}{1+e^z+\sqrt[4]{z}}\cdot(e^z + \frac{1}{4z^{3/4}})$
  3. Find the slope of $f(x) = \ln(xe^x)$ when $x = 2$.
     $f'(x) = \frac{1}{x} + 1 \quad\rightarrow\quad \text{Slope} = f'(2) = \frac{3}{2}$
  4. $\frac{d}{d\theta}[(\sec\ \theta)^{2\theta}] = (\sec\ \theta)^{2\theta}(2\ln(\sec\ \theta) + \frac{2\theta}{\sec\ \theta}\cdot\sec\ \theta\ \tan\ \theta)$
     $= (\sec\ \theta)^{2\theta}(2\ln(\sec\ \theta) + 2\theta\tan\ \theta)$
  5. $y = x^2\log_3 x \quad\rightarrow\quad \frac{dy}{dx} = 2x\log_3 x + x^2\frac{1}{x\ln\ 3}$

- Potpourri
  1. Find the velocity at 1 second if $s = f(t) = 4t^5 + 5t^3 - t + 2$ in meters at t seconds.
     $v = f'(t) = 20t^4 + 15t^2 - 1$
     Velocity at 1 second $= f'(1) = 20 + 15 - 1 = 34$ meters/second
  2. $\frac{d}{dx}[\ln(\sin\ \frac{\pi}{2})] = 0$
  3. Find $\frac{dy}{dx}$ for the curve $\frac{y}{\cos\ x} + 4y^2 = 18 + x$ at the point $(0, 2)$.
     $\frac{dy}{dx} = \frac{1}{17}$
  4. Find $\frac{dy}{dx}$ for the curve $x^2y + \arctan\ x = \frac{\pi}{2}$ at the point $(1, \frac{\pi}{4})$.
     $\frac{dy}{dx} = -\frac{\pi+1}{2}$

5. Find an equation of the tangent line of $g(x) = \dfrac{3+e^x}{4x-3}$ at the point $(0, -\frac{4}{3})$.

$y = -\frac{19}{9}x - \frac{4}{3}$