Math 124
Final Exam
June 12th, 2007

Name: _____

1. Your exam contains 7 questions and 9 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Note: To evaluate limits, proof by graph or table of values does not suffice for full credit.
4. If you need extra space, use the back of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing and scientific calculators are allowed.
6. Leave answers in exact form (as simplified as possible) or round to 4 decimal places.

Problem	Total Points	Score
1	12	
2	12	
3	16	
4	14	
5	24	
6	13	
7	9	
Total	100	

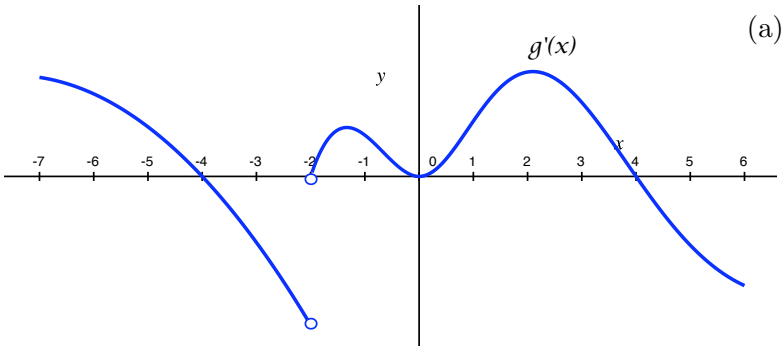
1. (12 pts.) Evaluate the following limits. Justify your answers.

(a) (4 pts.) $\lim_{x \to 2} \dfrac{\frac{2}{x^2} - \frac{1}{2}}{x-2}$

(b) (4 pts.) $\lim_{x \to \infty} \dfrac{\sqrt{x^2+1}}{\sqrt{9x^2-5}}$

(c) (4 pts.) $\lim_{x \to 2^-} \dfrac{|x-2|}{x^2-2x}$

2. (12 pts.) The following is a graph of $g'(x)$.



(a) (3 pts.) On what intervals is the function $g(x)$ increasing? On what intervals is $g(x)$ decreasing?

(b) (3 pts.) Assuming the function g is defined for all values in $[-7, 6]$, does g have local extreme values? If so, at what x-values does g have a local maximum? At what x-values does g have a local minimum?

(c) (3 pts.) For what x-values is the function g concave up? For what x-values is g concave down?

(d) (3 pts.) What are the x-coordinates of the inflection points of g?

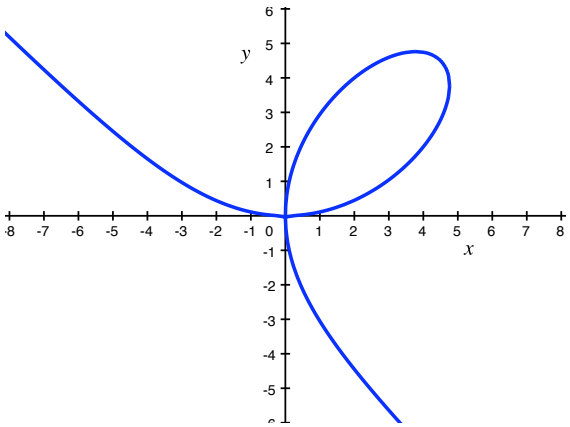
3. (16 pts.) Evaluate the following. You do not need to simplify your answer.

(a) (5 pts.) $\frac{d}{dt}[3t + 2\sqrt[3]{\arctan(3t)} - e^{\pi+1}]$

(b) (5 pts.) $\frac{d}{dx}\left[\frac{\sin^2(4x)}{\cos(x^3)}\right] = ?$

(c) (6 pts.) If $g(t) = (1-t)^{e^t}$, what is $g'(t)$?

4. (14 pts.) The curve shown below is $x^3 + y^3 = 9xy$.



(a) (2 pts.) Verify that the point $(2, 4)$ is on the curve.

(b) (7 pts.) Find the equation of the tangent line of the curve at the point $(2, 4)$.

(c) (5 pts.) Let $P = (x_1, 4.1)$ be a point on the curve near the point $(2, 4)$. Using part (b) and linear approximation, estimate x_1.

5. (24 pts.) Let $f(x) = e^{2x}\sin(2x)$ on the domain $[-1, 2]$.

(a) (6 pts.) What are the critical numbers of f in the domain?

(b) (4 pts.) For what intervals in the domain is f increasing? For what intervals is f decreasing?

(c) (5 pts.) For what intervals in the domain is f concave up? For what intervals is f concave down?

(d) (5 pts.) Find the local maximum and minimum values of f.

(e) (4 pts.) What are the absolute maximum and minimum values of f?

6. (13 pts.) The product of 2 positive numbers is 243. How small can the sum of one of the numbers plus the cube of the other number be? Show your work and justify that your solution is an absolute minimum.

7. (9 pts.)

$$h(t) = \begin{cases} t^2 - t - 2c & \text{for } t \geq -1 \\ -(t+2)^3 + c & \text{for } t < -1 \end{cases}$$

(a) (4 pts.) For what value of c is the function h continuous on $(-\infty, \infty)$?

(b) (5 pts.) Given the value of c from part (a), is h differentiable at $t = -1$? Justify your answer.