**Math 124**
**Exam 2 Solutions**

1. (55 pts.) Evaluate the following.

   (a) (10 pts.) For $g(x) = \frac{2\ln(x)+6}{-x^2+3x}$, find $g'(x)$.

   $$g'(x) = \frac{(-x^2+3x)(\frac{2}{x})-(2\ln(x)+6)(-2x+3)}{(-x^2+3x)^2} \qquad \text{(Quotient Rule)}$$

   (b) (7 pts.) $\frac{d}{dx}[\ln(e^2-1)] = ?$

   Since $\ln(e^2-1)$ is a constant, $\frac{d}{dx}[\ln(e^2-1)] = 0$.

   (c) (10 pts.) For $f(t) = 3^{e^t}$, find $f''(t)$.

   $$f'(t) = 3^{e^t} \cdot \ln 3 \cdot e^t$$

   $$f''(t) = \ln 3(3^{e^t}\ln 3 \cdot e^t \cdot e^t + 3^{e^t} \cdot e^t) \qquad \text{(Product Rule)}$$

   (d) (8 pts.) Find the 37th derivative of $\sin 5x$.

   Consider the first few derivatives of $\sin 5x$.

   1st: $\frac{d}{dx}(\sin 5x) = 5\cos 5x$
   2nd: $\frac{d}{dx}(5\cos 5x) = -5^2\sin 5x$
   3rd: $\frac{d}{dx}(-5^2\sin 5x) = -5^3\cos 5x$
   4th: $\frac{d}{dx}(-5^3\cos 5x) = 5^4\sin 5x$

   The power of 5 keeps increasing, so by the 37th derivative, the coefficient is $5^{37}$. Also by the 37th derivative, the derivative has cycled back to a positive $\cos 5x$. So, the 37th derivative is $5^{37}\cos 5x$.

   (e) (10 pts.) $\frac{d}{d\theta}[4\sec^2(5+2\theta)] = ?$

   $$\frac{d}{d\theta}[4\sec^2(5+2\theta)] = 4 \cdot 2\sec(5+2\theta) \cdot \sec(5+2\theta)\tan(5+2\theta) \cdot 2$$
   (Chain rule $\times 2$)

   (f) (10 pts.) Find $\frac{dy}{dt}$ for $y = (t+2)^{\tan t}$.

   $$\ln y = \ln(t+2)^{\tan t} = (\tan t)\ln(t+2)$$

   Differentiating both sides with respect to $t$:
   $$\frac{1}{y} \cdot \frac{dy}{dt} = \sec^2 t \cdot \ln(t+2) + \tan t\left(\frac{1}{t+2}\right)$$

$$\Rightarrow \quad \frac{dy}{dt} = (t+2)^{\tan\, t}(\sec^2 t \cdot \ln(t+2) + \tan\, t(\tfrac{1}{t+2}))$$

2. (15 pts.) The amount of water in a reservoir in millions of gallons is given by $w = f(t) = \sqrt{300t - 5t^2}$ on a given day $t$ with $0 \le t \le 60$.

   (a) (5 pts.) How much water is in the reservoir on day 50?

   $f(50) = \sqrt{300(50) - 5(50^2)} = 50$

   $\Rightarrow$ There was 50 million gallons in the reservoir on day 50.

   (b) (10 pts.) Is water flowing into or out of the reservoir on day 50 and at what rate? (Include units on the rate of water flow.)

   $$f'(t) = \frac{300 - 10t}{2\sqrt{300t - 5t^2}}$$
   $$\Rightarrow \quad f'(50) = \frac{300 - 10(50)}{2\sqrt{300(50) - 5(50^2)}} = -2$$

   Since the derivative is negative, the water is flowing out of the reservoir on day 50 at a rate of 2 million gallons per day.

3. (15 pts.) $f(x) = \arccos x = \cos^{-1} x$

   (a) (5 pts.) For what $x$-value is $f(x) = \frac{\pi}{3}$ ?

   If $f(x) = \frac{\pi}{3}$, then $\arccos x = \frac{\pi}{3} \quad \Rightarrow \quad x = \cos\frac{\pi}{3} = \frac{1}{2}$.

   (b) (10 pts.) Find an equation for the tangent line of $f$ at the $x$-value found in part (a).

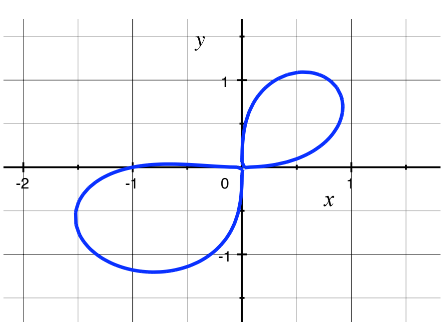
   We first need to find the slope of $f$ when $x = \frac{1}{2}$.

   $$f'(x) = -\frac{1}{\sqrt{1-x^2}} \quad \Rightarrow \quad f'(\tfrac{1}{2}) = -\frac{1}{\sqrt{1-(\frac{1}{2})^2}} = -\frac{1}{\sqrt{\frac{3}{4}}} = -\frac{2}{\sqrt{3}}$$

   The tangent line has slope $-\frac{2}{\sqrt{3}}$ and goes through the point $(\frac{1}{2}, \frac{\pi}{3})$.

   Equation: $y - \frac{\pi}{3} = -\frac{2}{\sqrt{3}}(x - \frac{1}{2})$

4. (10 pts.) Find the slope of the curve $x^3 + (x^2 + y^2)^2 = 4xy$ at the point $(-1, 0)$.




$$\frac{d}{dx}(x^3 + (x^2 + y^2)^2) = \frac{d}{dx}(4xy)$$
$$3x^2 + 2(x^2 + y^2)(2x + 2y\frac{dy}{dx}) = 4y + 4x\frac{dy}{dx}$$

Plugging in $x = -1$ and $y = 0$:

$$3(-1)^2 + 2(1+0)(-2+0) = 0 - 4\frac{dy}{dx}$$
$$-1 = -4\frac{dy}{dx}$$

$$\Rightarrow \quad \text{Slope} = \frac{dy}{dx} = \frac{1}{4}$$