

Math 151 Quiz #7 Answers

1. Taking the natural log of each side and simplifying: $\ln y = \sin x \cdot \ln(\ln(x))$

Implicit Differentiation (Using product rule and chain rule on the right side):

$$\frac{dy}{dx} = (\ln x)^{\sin x} [\cos x \cdot \ln(\ln x) + \sin x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}]$$

2. (a) The swallow is at rest when the velocity is equal to zero. $\Rightarrow 3t^2 - 18t + 15 = 0$
 $\Rightarrow 3(t - 5)(t - 1) = 0$

At Rest: $t = 1$ and $t = 5$ seconds

The swallow is moving in the positive direction when the velocity is positive. $\Rightarrow 3(t - 5)(t - 1) > 0$

Moving in the Positive Direction: $0 < t < 1$ and $t > 5$ seconds

Here are a couple of ways to determine where $v > 0$:

- Note that the graph of $v = 3t^2 - 18t + 15$ is a parabola that opens upward. Since it has zeros at $t = 1$ and $t = 5$, it is positive for $0 \leq t < 1$ and $t > 5$.
- If $0 \leq t < 1$, then both of the factors $t - 1$ and $t - 5$ are negative, so $v > 0$.
If $1 < t < 5$, then the factor $t - 1$ is positive, but the factor $t - 5$ is negative, so $v < 0$.
If $t > 5$, then both of the factors $t - 1$ and $t - 5$ are positive, so $v > 0$.

- (b) Acceleration = $a = 6t - 18$

Note that $a > 0$ (positive) for $t > 3$ and $a < 0$ (negative) for $0 < t < 3$.

Speeding up: $1 < t < 3$ (v and a are both negative), $t > 5$ (v and a are both positive)

- (c) Note: Using the position function, we can see (surprisingly) that the position at time 0 is 0 feet and the position at time 2 is 2 feet.

$$\text{Average velocity} = \frac{\Delta s}{\Delta t} = \frac{\text{Final Position} - \text{Initial Position}}{\text{Change in Time}} = \frac{2-0}{2-0} = 1 \text{ ft/sec}$$