

Math 151 Quiz #3 Answers

1. Horizontal Asymptote: $y = 3$ (This is obtained by looking at the limit of $f(x)$ as $x \rightarrow \infty$ and $-\infty$.)

Vertical Asymptote: $x = 1$ ($f(x)$ simplifies to $\frac{3(x-2)(x+2)}{(x+2)(x-1)} = \frac{3(x-2)}{x-1}$ so it has a V.A. at $x = 1$ and a hole at $x = -2$.)

2. $\lim_{t \rightarrow \infty} \frac{t^6 + 2t + 1}{1 - 2t^5} = -\infty$

Since this is a limit at infinity of a rational function, we can use the degree of the numerator and denominator to help us determine the limit. (Note: This only works when we are looking at $t \rightarrow \infty$ or $-\infty$.)

The limit is infinite ($+\infty$ or $-\infty$) since the degree of the numerator is larger than the degree of the denominator.

As $t \rightarrow \infty$, the numerator $t^6 + 2t + 1 \rightarrow \infty$ (positive), and the denominator $1 - 2t^5 \rightarrow -\infty$ (negative).

3. Using either definition, you should get that $g'(1) =$ slope of $g(x)$ at $x = 1 = -3$

$$\begin{aligned}\text{Definition 1: } g'(1) &= \lim_{x \rightarrow 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 - 5x + 2 - (-2)}{x - 1} \\ &= \lim_{x \rightarrow 1} x - 4 \quad (\text{Simplified}) \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{Definition 2: } g'(1) &= \lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h} = \lim_{h \rightarrow 0} \frac{(1+h)^2 - 5(1+h) + 2 - (-2)}{h} \\ &= \lim_{h \rightarrow 0} h - 3 \quad (\text{Simplified}) \\ &= -3\end{aligned}$$

4. Velocity of the bird at 5 seconds = Slope of the position function at 5 seconds
 $= -\frac{1}{2}$ or -0.5 ft/sec