Math 151 Quiz #10 Answers

1. $f'(x) = 5e^{\sin x} \cdot \cos x$ Note that the $f'(x)$ is defined for all x-values.

To find critical numbers, we must find values for which $f'(x) = 0$.

$$5e^{\sin x} \cdot \cos x = 0 \quad \Rightarrow \quad \cos x = 0 \qquad \text{since } 5e^{\sin x} > 0 \text{ for all } x.$$

$$\Rightarrow \quad x = \frac{\pi}{2} \text{ or } x = \frac{3\pi}{2}$$

$$\boxed{\text{So, the critical numbers are } x = \frac{\pi}{2} \text{ or } x = \frac{3\pi}{2}.}$$

2. **Critical Numbers:**
First, we need to find the critical numbers in the domain.

$$f'(x) = \frac{(x^2+4)(4)-4x(2x)}{(x^2+4)^2} \quad \text{(Quotient Rule)}$$
$$= \frac{-4x^2+16}{(x^2+4)^2}$$

Note that this is defined for all x-values since $x^2 + 4$ is always positive (never zero).

So, we need to find values for which $\frac{-4x^2+16}{(x^2+4)^2} = 0. \quad \Rightarrow \quad -4x^2 + 16 = 0 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$

Note that in our domain, the only critical number is $x = 2$.

Evaluating $f(x)$ at the endpoints and the critical numbers:

$f(0) = 0$ $f(2) = \frac{8}{8} = 1$ $f(6) = \frac{24}{40} = \frac{3}{5}$

Absolute Maximum Value: 1	**Absolute Minimum Value:** 0

3. (a) $f(x)$ is increasing when $f'(x)$ is positive $\boxed{\rightarrow \quad x < -2, \quad 1 < x < 3.4 \text{ (approx.)}, \quad x > 3.4}$

(b) $f(x)$ has a $\boxed{\text{local maximum at } x = -2}$, since $f'(x)$ switches from positive to negative at $x = -2$.

(c) $f(x)$ is concave up when $f''(x)$ is positive. Note: We can find where $f''(x)$ is positive, by looking finding values for which $f'(x)$ has positive slope.

So, $f(x)$ is concave up when $\boxed{-1 < x < 2, \text{ and } \quad x > 3.4.}$