Math 151
Final Exam
December 9th, 2009

Name: ________________________________

1. Your exam contains 9 questions and 8 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive partial credit unless all work is clearly shown. If in doubt, ask for clarification. Note: To evaluate limits, proof by graph or table of values does not suffice for full credit.
4. Put a box around your final answer where applicable.
5. Leave answers in exact form (as simplified as possible).
6. If you need extra space, attach an extra sheet to the back of the exam and clearly indicate this.
7. You are allowed one 8.5×11 sheet of handwritten notes (both sides).
8. You may use a calculator for this exam, but I will not give credit for work done solely on a calculator (aside from arithmetic).

Problem	Total Points	Score
1	20	
2	11	
3	12	
4	6	
5	8	
6	13	
7	13	
8	9	
9	8	
Total	100	

1. (20 pts.) Evaluate the following limits. Justify your answers.
If the limit is infinite, indicate whether it is $+\infty$ or $-\infty$.

(a) (5 pts.) $\lim_{x\to 2} \frac{\frac{1}{2}-\frac{2}{x^2}}{x-2}$

(b) (5 pts.) $\lim_{t\to\infty} \frac{\sqrt{t^2+1}}{\sqrt{9t^2-5}}$

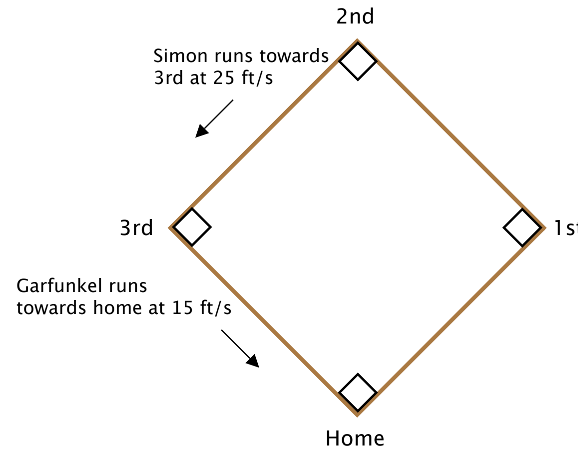
(c) (5 pts.) $\lim_{x\to 3^-} \frac{|x-3|}{x^2-6x+9}$

(d) (5 pts.) $\lim_{\theta\to 0} \frac{\tan^2\theta}{\sin\theta}$

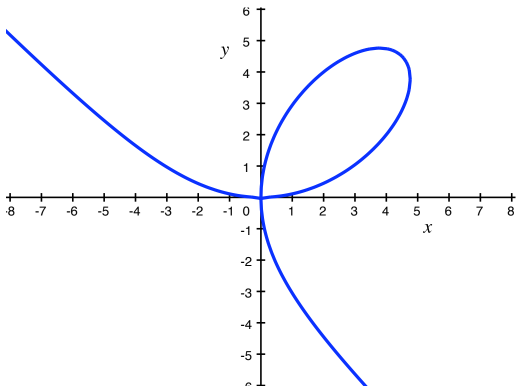
2. (11 pts.) During a baseball game, Simon is on second base and Garfunkel is on third base. When the batter hits the ball, Simon and Garfunkel simultaneously start running toward the next bases. Simon runs at a speed of 25 feet/second and Garfunkel runs at a speed of 15 feet/second.

How fast is the **distance between them** changing when Simon has run 50 feet and Garfunkel has run 30 feet? Is the distance between them increasing or decreasing?

(Note: The sides of a baseball diamond are 90 feet in length.)



3. (12 pts.) The curve shown below is $x^3 + y^3 = 9xy$.



(a) (1 pt.) Verify that the point $(2, 4)$ is on the curve.

(b) (7 pts.) Find an equation of the tangent line of the curve at the point $(2, 4)$.

(c) (4 pts.) Suppose $P = (x_1, 4.1)$ is the point on the curve near the point $(2, 4)$. Using part (b) and linear approximation, estimate x_1.

4. (6 pts.) For $y = (\arcsin t)^{3t}$, find $\dfrac{dy}{dt}$.

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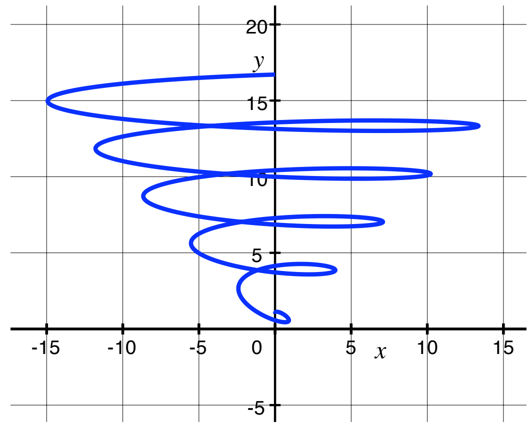
5. (8 pts.) Consider $h(t) = \begin{cases} 2t^2 + 2t + c & \text{for } t \geq 1 \\ \sqrt{24t - 20} + 3c & \text{for } t < 1 \end{cases}$

(a) (4 pts.) Find the value of c for which the function h is **continuous** at $t = 1$.

(b) (4 pts.) Given the value of c from part (a), is h **differentiable** at $t = 1$? (Justify your answer.)

6. (13 pts.) The squiggly curve below is described by the equations $x(t) = t\sin(2t)$, $y(t) = t + \cos(2t)$ for $0 \leq t \leq 5\pi$.



(a) (7 pts.) Find the slope of the tangent line to the curve at the point at which $t = \pi$.

(b) (6 pts.) Find **all** values of t in the interval $[0, \pi]$ at which the tangent line of the curve is horizontal. (Give exact values.)

7. (13 pts.) Consider $g(x) = \dfrac{(\ln x)^2}{x} + 12.$

(a) (10 pts.) Find the critical numbers of $g(x)$ and identify each as the location of a local maximum, local minimum, or neither by using the 1st or 2nd derivative test. (Give exact values.)

(b) (3 pts.) If the domain of $g(x)$ is restricted to the interval $[1, 20]$, find the **absolute** maximum and minimum **values** of $g(x)$ on the interval.

8. (9 pts.) Find the intervals of x for which the function $f(x) = 5x + e^{\frac{1}{3}x^3}$ is concave up.
(To clarify: The exponent of e is $\frac{1}{3}x^3$.)

9. (8 pts.) Find the positive number such that the the **sum of the number and its reciprocal** is as small as possible. (Justify that your answer is an absolute **minimum** using methods from section 4.1 or 4.3.)