Math 151
Exam 2 Answers

1. Point: $(1, f(1)) \;=\; (1, \frac{\pi}{4})$

   Slope: $f'(x) = \frac{1}{1+(\frac{1}{x})^2}(-\frac{1}{x^2})$ $\qquad\qquad$ $f'(1) = -\frac{1}{2}$

   Equation of Tangent: $\boxed{y - \frac{\pi}{4} = -\frac{1}{2}(x-1)}$

2. (a) The turkey's speed is greater at $\boxed{\text{4 seconds}}$ since the slope of the tangent to the position function is steeper at 4 seconds. (Velocity at t = slope of position at t, Speed = |velocity|)

   (b) The velocity is positive when the slope of the position function is positive.
   $$\Rightarrow \boxed{0 < x < 3, \quad 5 < x < 7}$$

   (c) Note: The turkey changes direction at times $t = 3$ and $t = 5$.

   Distance travelled = Dist. travelled from 0 to 3 sec + Dist. travelled from 3 to 5 sec + Dist. travelled from 5 to 7 sec

   = 2 feet + 5 feet + 6 feet
   = $\boxed{\text{13 feet}}$

3. To find the linearization at $x = 0$, we need $h(5)$ and $h'(5)$.

   - $h(5) = \ln(2(5)-9) + \frac{12}{5-3} = 6$
   - $h'(x) = \frac{2}{2x-9} - \frac{12}{(x-3)^2}$ $\qquad\qquad$ $h'(5) = \frac{2}{2(5)-9} - \frac{12}{(5-3)^2} = -1$

   So, the linearization is $L(x) = -(x-5) + 6$ or $\boxed{L(x) = 11 - x.}$

   $$\Rightarrow \; h(4.9) \approx L(4.9) = 11 - 4.9 = \boxed{6.1}$$

4. (a) $\qquad f(30) = \sqrt{120(30) - 3(30^2)} = 30$

   $\Rightarrow$ There was $\boxed{\text{30 million gallons}}$ in the reservoir on day 30.

   (b) $\qquad f'(t) = \frac{120-6t}{2\sqrt{120t-3t^2}}$

   $$\Rightarrow \; f'(30) = \frac{120-6(30)}{2\sqrt{120(30)-3(30^2)}} = \boxed{-1 \text{ millions of gallons/day}}$$

   Since the derivative is negative, the water is $\boxed{\text{flowing out}}$ of the reservoir on day 30 at a rate of 1 million gallons per day.

5. $f'(x) = 6x^5e^x + x^6e^x$ Solving $f'(x) = 0 \Rightarrow 6x^5e^x + x^6e^x = 0$

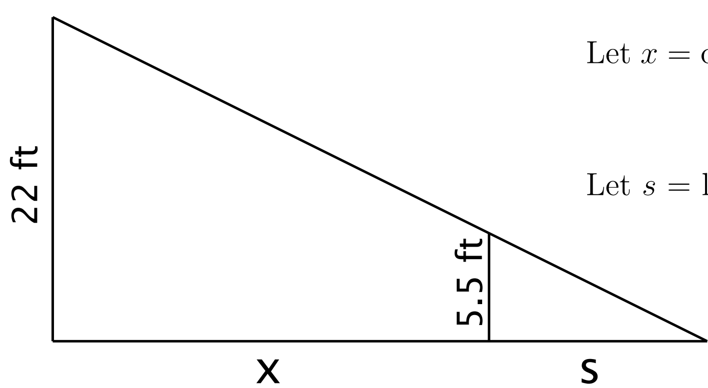
$x^5e^x(6+x) = 0 \Rightarrow \boxed{x = 0,\ x = -6}$

6. Consider the first few derivatives of $7\cos 2x$.

1st: $\frac{d}{dx}(7\cos 2x) = 7(-2\sin 2x)$
2nd: $\frac{d}{dx}(7(-2\sin 2x)) = 7(-2^2\cos 2x)$
3rd: $\frac{d}{dx}(7(-2^2\cos 2x)) = 7(2^3\sin 2x)$
4th: $\frac{d}{dx}(7(2^3\sin 2x)) = 7(2^4\cos 2x)$

The power of 2 keeps increasing, so by the 43rd derivative, the coefficient is 2^{43}. Also by the 43rd derivative, the derivative has cycled back to a positive $\sin 2x$. So, the 43rd derivative is $\boxed{7 \cdot 2^{43}\sin 2x.}$

7. (a) We have the following diagram:



Let x = distance between the woman and the streetlight

Let s = length of shadow

Unknown: $\frac{ds}{dt}$ when $x = 10$

Known: $\frac{dx}{dt} = 6$ ft/sec

Equation: Using similar triangles, we have that $\frac{x+s}{22} = \frac{s}{5.5}$

We can simplify this equation to $s = \frac{1}{3}x$.

Differentiating with respect to time: $\frac{ds}{dt} = \frac{1}{3}\frac{dx}{dt}$

Plugging in known values: $\frac{ds}{dt} = \frac{1}{3}(6) = 2$

The rate of change of the shadow is $\boxed{2 \text{ ft/sec.}}$

(b) The length of the shadow is $\boxed{\text{increasing.}}$ (The rate of change of the shadow length is positive.)