Math 151
Exam 1 Answers

1. (a) $\lim_{t\to 2^-} \frac{3t-6}{t^2-4t+4} = \lim_{t\to 2^-} \frac{3}{t-2}$ (Dividing by a common factor of $t-2$.)

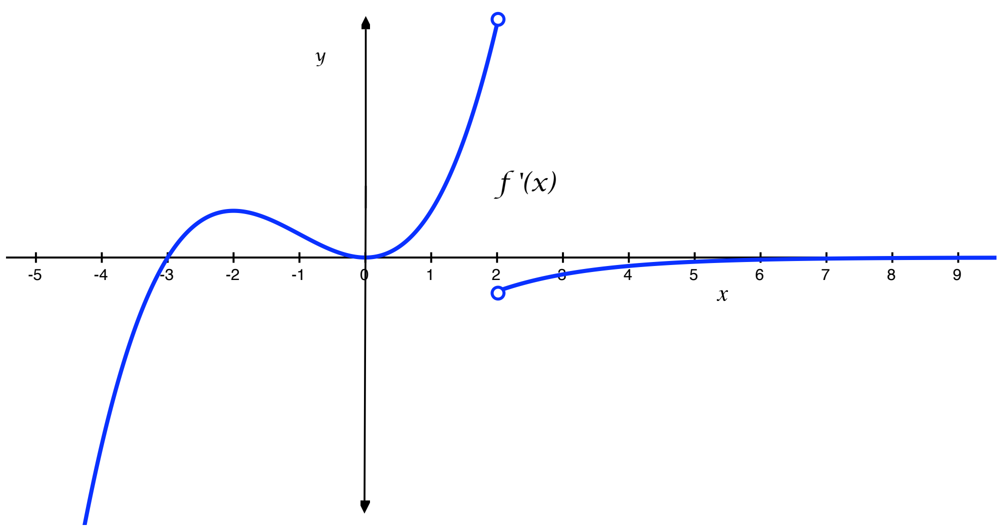
   As $t \to 2^-$, the numerator approaches 3 and the denominator is approaching 0. $\Rightarrow$ The limit is infinite. Since the numerator is positive and the denominator is negative (since $t < 2$), $\lim_{t\to 2^-} \frac{3t-6}{t^2-4t+4} = \lim_{t\to 2^-} \frac{3}{t-2} = \boxed{-\infty}$.

   (b) $\lim_{x\to -\infty} \frac{-3x^5-x^3+1}{9x^5+2x^2-x} = \boxed{-\frac{1}{3}}$ since the degree of the numerator and denominator are equal for this limit at infinity. (Ratio of coefficients)

   (c) $\lim_{x\to 3} \arcsin(\frac{x-3}{2x-6}) = \lim_{x\to 3} \arcsin(\frac{1}{2}) = \boxed{\frac{\pi}{6}}$ (Dividing by a common factor of $x-3$.)

   (d) $\lim_{r\to -3^+} \frac{e^r}{2r+2} = \boxed{-\frac{e^{-3}}{4}}$ (Evaluating this limit at $r=-3$.)

2. Note that $f'(x) = 0$ when $x = -3$ and $x = 0$.....



3. Need:

   1) Slope: The slope of the line is $m = 6$ since it is parallel to $y = 6x + 1$.

   2) Point: To find the point of tangency, we need to find the point at which $y = 16x^4 - 2x + 3$ has slope 6. $\Rightarrow$ $64x^3 - 2 = 6$ $\Rightarrow$ $x = \frac{1}{2}$

   So, the point is $(\frac{1}{2}, 3)$. (To find the y-value, evaluate $16x^4 - 2x + 3$ at $x = \frac{1}{2}$.)

   Equation: $\boxed{y - 3 = 6(x - \tfrac{1}{2}) \quad \text{or} \quad y = 6x}$

4. Using the product rule:
$$\frac{dy}{dt} = \boxed{2t^{-1/2}e^t \ + \ 4\sqrt{t}e^t \ + \ \pi}$$

5. (a) Note that $-\frac{1}{2}x^4 + 4x$ is continuous for all values $x > -1$.

   Note that $\frac{3}{x^2} - 2$ is continuous for all values $x < -1$. (The only value for which it may have a problem is at $x = 0$, which is not in this piece's domain.)

   So, the only value for which this function could be discontinuous is at $x = -1$.

   Since $\lim_{x\to -1+} h(x) = \lim_{x\to -1+} -\frac{1}{2}x^4 + 4x \ = \ -\frac{1}{2}(-1)^4 + 4(-1) = -4.5$

   and $\lim_{x\to -1-} h(x) = \lim_{x\to -1-} \frac{3}{x^2} - 2 \ = \ \frac{3}{(-1)^2} - 2 = 1$, there is a $\boxed{\text{jump discontinuity at } x = -1.}$

   (b) Since there are no infinite discontinuities (see part a), there are no vertical asymptotes.

   To find horizontal asymptotes:
   Since $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} -\frac{1}{2}x^4 + 4x \ = -\infty$,
   and $\lim_{x\to-\infty} h(x) = \lim_{x\to-\infty} \frac{3}{x^2} - 2 \ = \ -2$, there is only one horizontal asymptote,

   which is $\boxed{y = -2.}$

   (c) No, since $h(x)$ is not continuous at $x = -1$ (see part a), it is not differentiable at $x = -1$.

   (d) Taking the derivative of each function on its domain:
$$\boxed{h'(x) = \begin{cases} -\frac{6}{x^3} & \text{if } x < -1 \\ -2x^3 + 4 & \text{if } x > -1 \end{cases}}$$
   Note that the derivative is undefined at $x = -1$.

6.
$$\begin{aligned}
\lim_{h\to 0} \frac{g(x+h) - g(x)}{h} &= \lim_{h\to 0} \frac{3(x+h)^2 + \frac{1}{x+h} - (3x^2 - \frac{1}{x})}{h} \\
&= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} + \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h\to 0} 6x + 3h + \lim_{h\to 0} \frac{1}{h} \cdot \frac{-h}{x(x+h)} \qquad \text{(Common denominator and simplifying)} \\
&= \lim_{h\to 0} 6x + 3h + \lim_{h\to 0} \frac{-1}{x(x+h)} \\
&= 6x - \frac{1}{x^2}
\end{aligned}$$