Math 151 Quiz #9

1. Consider the curve described by the parametric equations $x(t) = \sin(4t)$ and $y(t) = e^t$.

   (a) $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)} = \dfrac{e^t}{\cos(4t)\cdot 4} = \dfrac{e^t}{4\cos(4t)}$

   To find $\dfrac{dy}{dx}$ at the point (0,1), we need to know the value of t that gives the x-value of 0 and the y-value of 1.

   $$\Rightarrow \quad 0 = \sin(4t) \qquad\qquad 1 = e^t$$

   There are multiple values ($t = k\pi$, for an integer k) that would satisfy the first equation, but only one value ($t = 0$) satisfies the second equation. $\Rightarrow$ $t = 0$

   So, $\dfrac{dy}{dx}$ at (0,1) is given by $\left.\dfrac{dy}{dx}\right|_{t=0} = \dfrac{y'(0)}{x'(0)} = \dfrac{e^0}{4\cos(0)} = \dfrac{1}{4}$.

   (b) The curve will have a vertical tangent if $x'(t) = 0$ with $y'(t) \neq 0$.

   Note that $y'(t) = e^t$ is never zero.

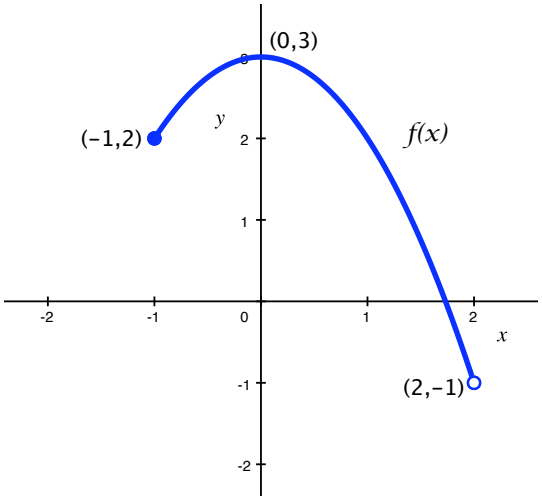
   $$x'(t) = 0 \quad \Rightarrow \quad 4\cos(4t) = 0 \quad \Rightarrow \quad \cos(4t) = 0$$

   Since $\cos\theta = 0$ when $\theta = \dfrac{\pi}{2} + k\pi$ or $\theta = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots,$ we have that

   $$4t = \frac{\pi}{2} + k\pi \quad \Rightarrow \quad t = \frac{\pi}{8} + \frac{k\pi}{4} \quad \text{or} \quad t = \pm\frac{\pi}{8}, \pm\frac{3\pi}{8}, \pm\frac{5\pi}{8}, \ldots.$$

   So, the function has vertical tangents when $t = \dfrac{\pi}{8} + \dfrac{k\pi}{4}$ or $t = \pm\dfrac{\pi}{8}, \pm\dfrac{3\pi}{8}, \pm\dfrac{5\pi}{8}, \ldots.$

2. The graph is given below.



Absolute Max. Value: 3 (Occurs at $x = 0$)

Absolute Min. Value: None (The function does not attain the value -1, since there is a hole.)

Local Max. Value: 3 (Occurs at $x = 0$)

Local Min. Value: None (Local extreme values do not occur at the endpoints given our definition.)