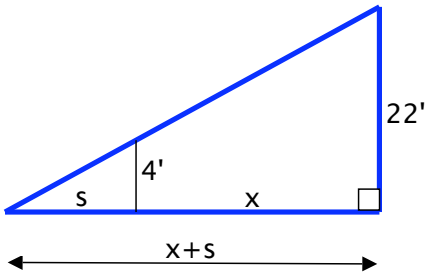
Math 124 Quiz #8 Solutions

1. Consider the following diagram:



Quantities:
s = Length of shadow
x = Distance between person and streetlight

Rates: $\frac{ds}{dt} = ?$ when $x = 8$
$\frac{dx}{dt} = 3$ ft/sec

Equation: Using similar triangles, we have that $\frac{s}{4} = \frac{x+s}{22}$ (You may have a different set of ratios, but it should simplify to the same result.)

$$\Rightarrow \quad 22s = 4x + 4s$$
$$\Rightarrow \quad 18s = 4x$$

Differentiating both sides with respect to time: $18\frac{ds}{dt} = 4\frac{dx}{dt}$

Plugging in known values at the time at which $x = 8$: $18\frac{ds}{dt} = 4(3)$
$$\Rightarrow \quad \frac{ds}{dt} = \frac{12}{18} = \frac{2}{3} \text{ ft/sec}$$

So, the length of shadow is growing at a rate of $\frac{2}{3}$ ft/sec.

2. (a) The linearization of $f(x)$ at $a = 3$ is the tangent line function of $f(x)$ at $a = 3$.

Point of Tangency: $(3, f(3)) = (3, 6)$ since $f(3) = 6 + 4\ln(6-5)$
$= 6 + 4\ln(1)$
$= 6 + 0 \ = \ 6$

Slope: $f'(x) = 0 + 4 \cdot \frac{1}{2x-5} \cdot 2 = \frac{8}{2x-5}$

Slope at $a = 3$: $m = f'(3) = \frac{8}{2(3)-5} = 8$

Linearization: $L(x) = 8(x-3) + 6$ or $L(x) = 8x - 18$ (You can use either in part (b).)

(b) $f(3.1) \approx L(3.1) = 8(3.1-3) + 6$
$= 8(.1) + 6$
$= 6.8$

So, $f(3.1)$ is approximately 6.8.